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The simplest chaotic Lotka-Volterra system with reflection, rotation, and inversion symmetries

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ABSTRACT

This study presents the simplest known three-species Lotka–Volterra system capable of exhibiting chaotic dynamics. The model is constructed with nonlinear growth and mortality terms defined as products of population densities and quadratic functions of species concentrations, capturing essential ecological nonlinearities in a minimal framework. Unlike many classical three-species Lotka–Volterra models, which typically exhibit only stable or periodic behavior, this system displays rich dynamical behaviors, including chaos, under specific parameter regimes and seven terms. Bifurcation analysis and Lyapunov exponent calculations confirm transitions between periodic oscillations and chaotic attractors. Notably, the chaotic attractor possesses a rare combination of reflection, rotation, and inversion symmetries, despite the system's structural simplicity. These results demonstrate that even the most minimal Lotka–Volterra formulations can generate multiple symmetric chaotic attractors, establishing a new benchmark in the study of simple yet chaotic ecological models.

1. Introduction

The Lotka-Volterra system, originally developed to describe predator-prey interactions and competitive species dynamics, remains one of the foundational models in theoretical ecology [1]. Its classical form, based on linear interaction terms, provides valuable insights into population oscillations and coexistence [1]. However, real ecological systems often exhibit nonlinear interactions that are not adequately captured by the original model [2]. Extensions incorporating nonlinear functional responses, higher-order interactions, and more complex feedbacks have been developed to address these limitations [3]. Despite their simplicity, Lotka-Volterra-type models continue to be widely used due to their analytical tractability and ability to reveal fundamental ecological principles [4]. Moreover, the importance of Lotka–Volterra models extends well beyond ecology because the underlying principle, agents competing for or sharing limited resources, applies universally to many complex systems like biology, economics, and energy systems [5,6].

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Chaotic dynamics in ecological models reflect the inherent unpredictability and complex temporal patterns observed in natural populations, which cannot be explained by simple periodic oscillations alone [7]. The exploration of chaos within Lotka-Volterra frameworks thus bridges classical ecological theory with modern nonlinear dynamics, providing a richer understanding of population fluctuations, species coexistence, and ecosystem stability under realistic, nonlinear interaction structures. For example, Ref. [8] studied a three-species Lotka-Volterra system and reported Hopf bifurcation and a cascade of period-doubling as parameters are varied, leading to chaotic attractors. In more recent work, Ref. [9] examined generalized Lotka-Volterra and May-Leonard biodiversity models, quantifying chaotic behavior via methods such as Hamming distance density, and showed that chaos arises also in higher-dimensional biodiversity contexts under modest interaction complexity.

Chaotic systems, characterized by their extreme sensitivity to initial conditions and unpredictability, have become a cornerstone in understanding complex dynamics across disciplines [10]. As highlighted in the literature on chaotic systems [11], a new model should meet at least one of the three criteria for scientific significance:

- It models an important unsolved natural problem,
- It exhibits novel dynamical behavior,
- It is simpler than all previously known systems with similar dynamics.

While chaotic dynamics have been previously observed in three-species Lotka–Volterra systems with nine or more terms [12–14], such models typically require additional nonlinear interactions and a larger number of parameters. In contrast, the system introduced here achieves chaos with only seven terms and two parameters, making it the structurally simplest known chaotic variant, thereby fulfilling the simplicity criterion.

Recent advancements in chaos theory have emphasized the role of symmetry in stabilizing or destabilizing these systems, offering insights into their structural robustness and functional adaptability. For instance, [15] highlight how symmetry-breaking mechanisms in chaotic systems can lead to emergent patterns, bridging the gap between theoretical models and real-world phenomena. The classification of symmetries, reflection, rotational, and inversion, provides a framework for analyzing the structural properties of chaotic systems. Reflection symmetry, where a system remains invariant under variable sign inversion ($x_i \rightarrow -x_i$) is prevalent in different systems as noted in [16]. Rotational symmetry, involving invariance under two variable sign ($(x_i, x_j) \rightarrow (-x_i, -x_j)$), is critical in modeling systems with angular momentum, such as fluid dynamics or celestial mechanics [17]. Inversion symmetry, where all variables are simultaneously reversed ($(x_1, x_2, x_3) \rightarrow (-x_1, -x_2, -x_3)$), is often observed in physical systems governed by conservation laws [18]. These symmetries not only dictate the system's response to perturbations but also influence the coexistence of multiple attractors [19]. By leveraging these symmetries, researchers can design models that balance complexity with predictability, enabling applications in diverse fields.

The practical applications of chaotic systems span from biological modeling to secure communication and hardware implementation. In neuroscience, chaotic models like the improved Hodgkin-Huxley [20], Hindmarsh-Rose [21], improved FitzHugh-Nagumo [22], and Chay [23], Wilson [24] neuron models have been instrumental in simulating the irregular firing patterns of neurons. These models capture the nonlinear interactions within neural networks, offering insights into epilepsy, brain oscillations, and information processing. In engineering, chaotic systems are used for image and signal encryption, where their unpredictability ensures robust security [25]. For example, [26] propose chaotic encryption algorithm that leverages the sensitivity of chaotic maps to scramble data efficiently. Additionally, implementing chaotic circuits has enabled real-time applications in secure communication, random number generation, and neuroscience [27]. In the context of symmetric chaotic systems, [28] introduced a chaotic encryption scheme that achieves high security and efficiency by utilizing rotational symmetry in phase space, making it robust against attacks.

The study is organized as follows. Section 2 introduces the general Lotka-Volterra framework and details the derivation of the simplest three-species chaotic system. Section 3 presents the fixed points analysis, including stability evaluation via eigenvalue computation. In Section 4, bifurcation diagrams and Lyapunov exponents calculations are used to explore the transitions between dynamic regimes. Section 5 examines the symmetry properties of the system. Finally, Section 6 summarizes the main findings.

2. The simplest chaotic Lotka-Volterra system

This study presents the simplest chaotic Lotka-Volterra system featuring nonlinear interactions among three species. The model is constructed such that each species' growth and mortality rates are expressed as the product of its population density and a quadratic polynomial function of the densities of the species. This approach captures essential nonlinear characteristics, including saturating responses, threshold effects, and indirect ecological controls, phenomena frequently observed in natural populations but rarely incorporated simultaneously in simple three-species Lotka-Volterra models. The general formulation is represented as,

$$\frac{dx_i}{dt} = x_i.Q_i(X), i = 1, 2, 3. \tag{1}$$

where $X = [x_1, x_2, x_3]^T$ denotes the population density of species, and $Q_i(X)$ is a quadratic function dependent on the populations of the species i, x_i. Over the past decades, researchers have endeavored to identify minimal or simplified chaotic systems that retain the essential mechanisms generating chaos, facilitating both theoretical understanding and practical application [29–34]. To identify the simplest chaotic system within Formulation 1 that retains the capacity for complex dynamics, an extensive parametric exploration was performed. This involved iterative computational analysis across a wide range of parameter values to systematically reduce the model

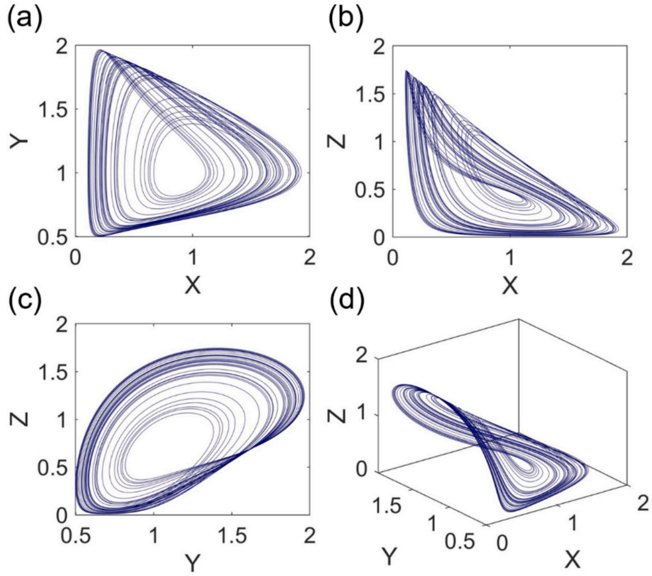

Fig. 1. The attractor of System (2) is shown in (a) x-y, (b) x-z, (c) y-z and (d) x-y-z planes when $(x_0, y_0, z_0) = [1, 1, 1]$ and $a = 0.4$ and $b = 2$. The figure shows that the system has chaotic attractor under these initial conditions and parameters.

to its simplest form that satisfying Ref. [11] Criterion (3) for publication of new chaotic systems. The outcome of this procedure is the three-species Lotka-Volterra system as,

$$\begin{aligned} \dot{x} &= -x(1 - y^2) \\ \dot{y} &= -y(ay^2 - z^2) \\ \dot{z} &= z(b - x^2 - y^2) \end{aligned} \tag{2}$$

When a and b are constant parameters. This system models a three-species food chain in which each population interacts through nonlinear terms characteristic of extended Lotka–Volterra dynamics. Each term in the system encapsulates biologically meaningful interactions that go beyond simple linear predator–prey relationships. In this model, the predator x preys on y, and its growth is stimulated only when the prey population (y) exceeds a threshold, modeled through a quadratic dependence. Species y functions as a consumer feeding on z, with its growth depending positively on z availability and negatively on its own density, reflecting intraspecific competition. The variable z represents a resource or primary producer that grows exponentially in the absence of consumers (x and y). So, z provides a continuous input to the food chain. Notably, x does not feed directly on z. However, its presence increases predation on y, which in turn imposes greater consumption pressure on z.

All growth and mortality terms are formulated as the product of the population and a quadratic expression in other species' densities, capturing key nonlinear features such as saturating responses, threshold effects, and indirect ecological control. The structure of the system and the form of the nonlinearities suggest the possibility of complex dynamics such as oscillations or chaotic trajectories. Such behavior is rarely observed in minimal three-species Lotka–Volterra-type models. The system shows chaotic behavior when $a = 0.4$ and $b = 2$. The chaotic attractor of System (2) is shown in Fig. 1 (a) xy, (b) xz, (c) yz surfaces and (d) xyz plane when $(x_0, y_0, z_0) = [1, 1, 1]$. To further characterize the system's dynamics, the Lyapunov exponents of the system are computed using the Wolf's algorithm. The results are [−0.86,0.00,0.06], where the presence of a positive exponent confirms chaotic behavior. Moreover, the mean value of the divergence along trajectories was found to be –0.8, confirming that the system is dissipative.

3. Fixed points analyses

Fixed points of the proposed system correspond to population states where all species' abundances remain constant over time. Identification of these equilibrium points involves solving the set of nonlinear equations derived from setting the right-hand side of each differential equation to zero as $\dot{x} = \dot{y} = \dot{z} = 0$. For the given model, this procedure yields a set of algebraic equations for the species densities as a function of the parameters a and b. When $a = 0.4$ and $b = 2$, the system has thirteen fixed points as $(0, 0, 0)$, $(0, \pm\sqrt{2}, \pm\sqrt{0.8})$, and $(\pm 1, \pm 1, \pm\sqrt{0.4})$. The Jacobian matrix of System (2) is as,

Table 1
The eigenvalues and the stability of the fixed points of System (2). The table shows that the system has unstable equilibria when $a = 0.4$ and $b = 2$.

(x, y, z)	Eigenvalues	Stability
$(0, 0, 0)$	$(-1.000, 0.000, 2.000)$	Unstable
$(0, \pm\sqrt{2}, \pm\sqrt{0.8})$	$(1.000, -0.800 \pm 2.400i)$	Unstable
$(\pm 1, \pm 1, \pm\sqrt{0.4})$	$(-1.358, 0.279 \pm 1.510i)$	Unstable

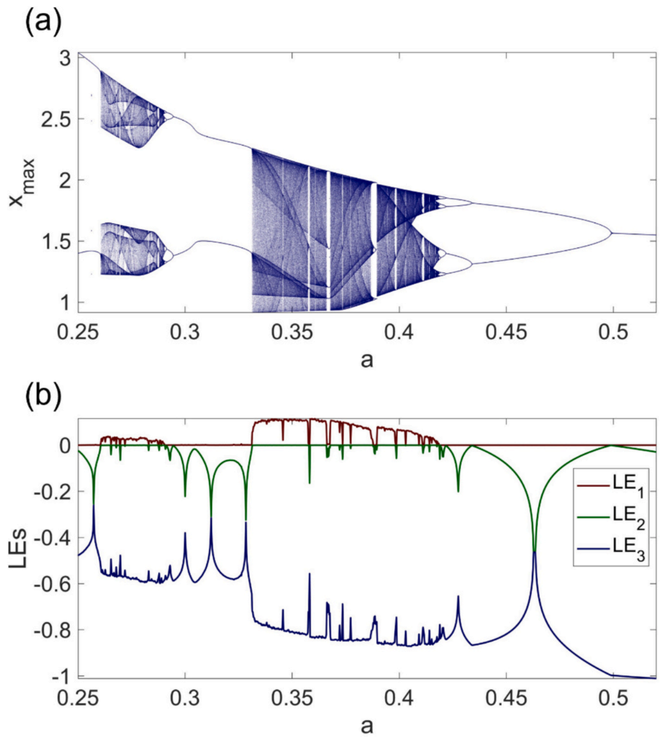

Fig. 2. The (a) bifurcation diagram and (b) Lyapunov exponents of System (2) as the parameter a increases when $b = 2$. The figures show that the system has periodic and chaotic attractors for different values of parameter a. Also, the Lyapunov exponents diagram confirms the bifurcation diagram, and whenever the bifurcation shows a chaotic attractor, the largest Lyapunov exponent is positive.

$$J = \begin{bmatrix} \frac{\partial \dot{x}}{\partial x} & \frac{\partial \dot{x}}{\partial y} & \frac{\partial \dot{x}}{\partial z} \\ \frac{\partial \dot{y}}{\partial x} & \frac{\partial \dot{y}}{\partial y} & \frac{\partial \dot{y}}{\partial z} \\ \frac{\partial \dot{z}}{\partial x} & \frac{\partial \dot{z}}{\partial y} & \frac{\partial \dot{z}}{\partial z} \end{bmatrix} = \begin{bmatrix} -(1-y^2) & 2xy & 0 \\ 0 & -(1.2y^2 - z^2) & 2yz \\ -2xz & -2yz & 2 - x^2 - y^2 \end{bmatrix} \quad (3)$$

The fixed points were classified by analyzing the eigenvalues of the Jacobian matrix evaluated at each equilibrium. The results are summarized in Table 1, where the stability nature of each fixed point is determined.

Examination of the eigenvalue spectra reveals that no stable fixed points exist within the phase space. Numerical investigations further confirm that trajectories initialized near these points do not converge but instead diverge or display chaotic responses.

4. Bifurcation and Lyapunov exponents

The dynamics of the system were further explored through bifurcation analysis to determine how changes in parameters a and b influence the qualitative behavior of species interactions. By systematically varying selected parameters' ranges, transitions between different dynamical regimes were identified. To quantify and confirm chaotic behavior within certain parameter regimes, the Lyapunov Exponents (LEs) was computed numerically using the Wolf algorithm [35]. In this method, small perturbations around a reference trajectory are tracked, and their exponential growth or decay rates are averaged over time, yielding numerical estimates of

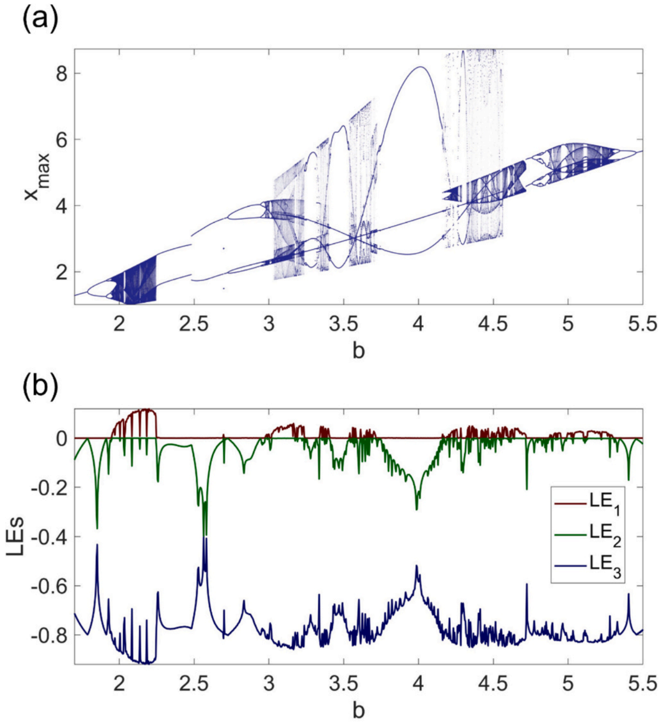

Fig. 3. The (a) bifurcation diagram and (b) Lyapunov exponents of System (2) as the parameter b increases when $a = 0.4$. The figures show that the system has periodic and chaotic attractors for different values of parameter b. Also, the Lyapunov exponents diagram confirms the bifurcation diagram, and whenever the bifurcation shows a chaotic attractor, the largest Lyapunov exponent is positive.

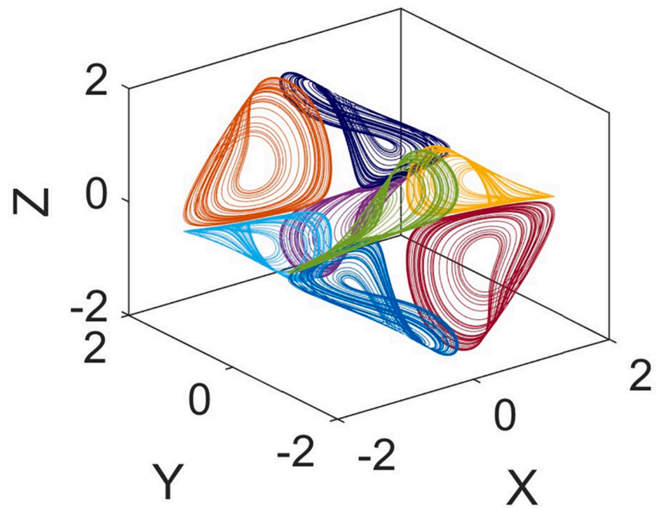

Fig. 4. The chaotic attractors of System (2) when $(x_0, y_0, z_0) = [\pm 1, \pm 1, \pm 1]$ and $a = 0.4$ and $b = 2$. These attractors are coexisting, and the system displays reflection, rotational, and inversion symmetries, demonstrating that multiple chaotic states exist simultaneously to preserve the underlying symmetrical structure of the system.

the Lyapunov spectrum. A positive largest Lyapunov Exponent (LLE) indicates sensitive dependence on initial conditions, chaotic dynamics, whereas zero or negative values correspond to periodic or stable behaviors, respectively.

In Fig. 2a, the bifurcation diagram shows that the variation in the parameter a leads to the emergence of complex attractors in different parameter ranges. Positive LLE values in Fig. 2b, correspond to the parameter regions in Fig. 2a reveals the presence of chaotic attractors.

In Fig. 3a, the bifurcation diagram illustrates how variation in the parameter b induces the formation of complex attractors across

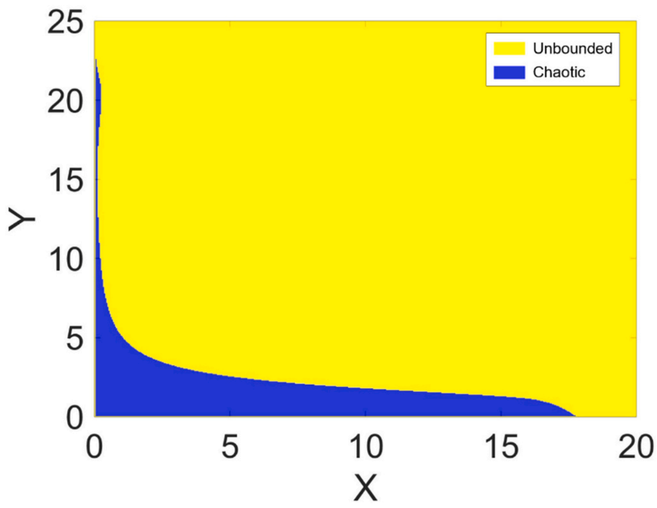

Fig. 5. Basins of attraction of System (2) when $z_0 = 1$ and $x_0, y_0 > 0$. The blue region corresponds to initial conditions that lead to chaotic dynamics, while the yellow region indicates initial conditions resulting in unbounded population growth or decline. The figure shows that the only bounded response of the system is chaotic attractor when $a = 0.4$ and $b = 2$. (For interpretation of the references to colour in this figure legend, the reader is referred to the web version of this article.)

distinct parameter intervals. Also, positive values of the largest Lyapunov exponent within these ranges, in Fig. 3b, confirm the presence of chaos.

5. Symmetry analysis

System (2) shows the rare phenomenon of coexistence of reflection, rotation, and inversion symmetries. This can be proved by the polarity balance under any of the variable reverse. With a detailed look to System (2), it can be seen that it remains the same under changing the variables in the form of $x \rightarrow \pm x$, $y \rightarrow \pm y$, $z \rightarrow \pm z$, showing its immunity against the polarity reverse indicating the coexistence of three regimes of symmetry.

Interestingly, the symmetry of the system does not generate symmetric attractor like those symmetric systems in [34,36], but gives coexisting attractors. However, this is also different from those chaotic systems with conditional symmetry [18,37], where the polarity balance needs an extra operation of offset boosting. As shown in Fig. 4, eight coexisting attractors are captured. These attractors are derived by considering different initial conditions as $(x_0, y_0, z_0) = [\pm 1, \pm 1, \pm 1]$. The basin of attraction of the attractor of System (2) when $z_0 = 1$ and $x_0, y_0 > 0$ is shown in Fig. 5. The figure illustrates that initial conditions represented by the blue region lead to chaotic trajectories, whereas those located within the yellow region result in unbounded population growth.

6. Conclusions

This study has presented the simplest three-species Lotka-Volterra system capable of exhibiting chaotic dynamics, meeting a key criterion for the publication of new chaotic systems. The formulation, grounded in nonlinear interaction terms expressed as quadratic functions of species densities, captures essential ecological processes such as saturating responses and threshold effects. Through extensive parametric exploration, the simplest chaotic form of the model was derived, containing only seven terms, making it the most structurally simplified chaotic Lotka-Volterra system known to date. Analysis of fixed points revealed the absence of stable equilibria. Bifurcation analysis and computation of Lyapunov exponents demonstrated that, depending on parameter values, the system exhibits periodic and chaotic dynamics. The attractors display reflection, rotation, and inversion symmetries within its structure. The findings highlight the critical role of nonlinearities and symmetries in shaping ecological interactions and suggest that even the simplest three-species model can exhibit rich dynamics. Also, it should be noted that in ecological modeling, population variables are inherently non-negative, and negative values lack physical meaning. This work contributes to a deeper understanding of population dynamics, offering a novel theoretical basis for exploring complex behaviors in ecological and other biological systems.

CRediT authorship contribution statement

Sajad Jafari: Writing – review & editing, Supervision, Conceptualization. **Atiyeh Bayani:** Writing – original draft, Software, Formal analysis. **Karthikeyan Rajagopal:** Visualization, Validation, Data curation. **Chunbiao Li:** Writing – original draft, Validation, Resources. **Julien Clinton Sprott:** Writing – review & editing, Supervision, Investigation.

Declaration of competing interest

The authors declare that they have no conflict of interest.

Data availability

No new data were created or analyzed in this study.

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