Exam 3

Print your name and section clearly on all five pages. (If you do not know your section number, write your TA's name.) Show all work in the space immediately below each problem. Your final answer must be placed in the box provided. Problems will be graded on reasoning and intermediate steps as well as on the final answer. Be sure to include units wherever necessary, and the direction of vectors. Each problem is worth 25 points. In doing the problems, try to be neat. Check your answers to see that they have the correct dimensions (units) and are the right order of magnitudes. You are allowed one 8½ x 11" sheet of notes and no other references. The exam lasts exactly 50 minutes.

(Do not write below)

SCORE:

Problem 1: __________

Problem 2: __________

Problem 3: __________

Problem 4: __________

TOTAL: __________
1. A laser beam is incident on a pair of slits separated by 0.12 mm.
   
   a. What is the slit-to-screen distance if the bright fringes are 5.0 mm apart and the laser light has a wavelength of 633 nm? (9 pts.)
   
   \[ \frac{\lambda}{d} = \frac{\mu}{L} \]
   
   \[ L = \frac{\mu \cdot d}{\lambda} = \frac{5 \times 10^{-3} \times 0.12 \times 10^{-3}}{633 \times 10^{-9}} \]
   
   \[ = 0.948 \text{ m} \]
   
   b. What will be the fringe spacing on the screen above with 480-nm light? (8 pts.)
   
   \[ \mu_0 = \frac{\lambda L}{d} = \frac{480 \times 10^{-9} \times 0.948}{0.12 \times 10^{-3}} \]
   
   \[ = 3.79 \times 10^{-3} \text{ mm} \]
   
   c. If the whole apparatus is immersed in water with an index of refraction of 1.33, what will be the fringe spacing on the screen for the 480-nm light above? (8 pts.)
   
   \[ \mu_0 = \frac{\lambda L}{nd} = \frac{480 \times 10^{-9} \times 0.948}{1.33 \times 0.12 \times 10^{-3}} \]
   
   \[ = 2.85 \times 10^{-3} \text{ mm} \]
2. An advanced civilization has developed a 1000-kg spaceship that goes, with respect to the galaxy, only 50 km/s slower than light.

   a. How long, according to the ship’s crew, does it take to cross the 100,000 light-year diameter of the galaxy? (7 pts.)

   \[
   \gamma = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}} = \sqrt{1 - \left(\frac{3 \times 10^8 - 5 \times 10^4}{3 \times 10^8}\right)^2} = 54.8
   \]

   \[
   \tau = \frac{L}{\gamma v} \approx \frac{L_0}{\gamma c} = \frac{100,000}{54.8} = 1826 \text{ yrs}
   \]

   **1826 yrs**

   b. What is the galactic diameter as measured in the ship’s frame of reference? (6 pts.)

   \[
   L = \frac{L_0}{\gamma} = \frac{100,000}{54.8} = 1826 \text{ ly-yr}
   \]

   **1826 ly-yr**

   c. What is the momentum of the spaceship in the frame of reference of the galaxy? (6 pts.)

   \[
   P = \gamma m v \approx 54.8 \times 10^3 \times 3 \times 10^8
   \]

   \[
   = 1.64 \times 10^{13} \text{ kg-m/s}
   \]

   **1.64 \times 10^{13} \text{ kg-m/s**

   d. What work must be done on the spaceship to accelerate it to this speed from rest? (6 pts.)

   \[
   W = \Delta KE = (\gamma - 1) mc^2
   \]

   \[
   = 53.8 \times 10^3 \times 9 \times 10^{16}
   \]

   \[
   = 4.84 \times 10^{21} \text{ J}
   \]

   **4.84 \times 10^{21} \text{ J**

3. A hydrogen discharge emits light with a wavelength of 103 nm.

   a. What principal quantum numbers are involved in this transition? (7 pts.)

   \[
   \frac{1}{\lambda} = R_H \left( \frac{1}{n^2} - \frac{1}{m^2} \right) \\
   \frac{1}{n^2} - \frac{1}{m^2} = \frac{1}{\lambda} R_H = \frac{1}{103 \times 10^{-9} \times 1.1 \times 10^{-7}} = \frac{8}{9} = 1 - \frac{1}{9} \\
   \Rightarrow n = 1, \ m = 3
   \]

   \[3 \rightarrow 1\]

   b. What is the energy of the emitted photons (in eV)? (6 pts.)

   \[E = hf = \frac{hc}{\lambda} = \frac{6.63 \times 10^{-34} \times 3 \times 10^8}{103 \times 10^{-9}} = 1.93 \times 10^{-18} \text{ J} = 12.1 \text{ eV}\]

   \[12.1 \text{ eV}\]

   c. If these photons are incident on a metal with a work function of 4.1 eV, what is the maximum energy of the ejected electrons? (6 pts.)

   \[K_{\text{max}} = E - \Phi = 12.1 - 4.1 = 8 \]

   \[8 \text{ eV}\]

   d. What is the minimum de Broglie wavelength of the electrons ejected above? (6 pts.)

   \[\lambda = \frac{h}{\sqrt{2mK}} = \frac{h}{\sqrt{2 \times 9.1 \times 10^{-31} \times 8 \times 1.6 \times 10^{-19}}} = 0.434 \text{ nm}\]

   \[4.34 \times 10^{-10}\]

a. How many electrons are in the L shell in the ground state of copper? (5 pts.)

\[ n = 2, \quad l = 0, \quad m_l = 0 \Rightarrow 2 \]

\[ l = 1, \quad m_l = 0, \pm 1 \Rightarrow 6 \]

\[ 2 + 6 = 8 \]

b. What is the orbital angular momentum of an electron in the 2p state? (5 pts.)

\[ 2p \Rightarrow l = 1 \]

\[ L = \sqrt{l(l+1)} \hbar = \sqrt{2} \times \frac{6.63 \times 10^{-34}}{2\pi} \]

\[ = 1.49 \times 10^{-34} \text{ m} \cdot \text{sec} \]

1.49 x 10^{-34} m/s

c. What is the energy (in eV) of the Kα characteristic x-ray emitted by copper? (5 pts.)

\[ \Delta E = \frac{3}{4} \times 13.6 (Z - 1)^2 \]

\[ = \frac{3}{4} \times 13.6 \times 2^2 = 8000 \text{ eV} \]

8000 eV

d. What is the wavelength of the x-ray calculated above? (5 pts.)

\[ \lambda = \frac{hc}{\Delta E} = \frac{hc}{c} = \frac{hc}{E} \]

\[ = \frac{6.63 \times 10^{-34} \times 3 \times 10^8}{8000 \times 1.6 \times 10^{-19}} \]

\[ = 1.55 \times 10^{-10} \text{ m} \]

0.155 nm

e. To what quantum state would you expect the outermost (valence) electron of copper in the 4s state to be excited by the absorption of a photon of the lowest possible energy? (5 pts.)

\[ \Delta l = \pm 1 \quad \Delta n = +1 \]

\[ n = 5, \quad l = 1 \Rightarrow 5p \]

5p