Exam 1

Print your name and section clearly on all five pages. (If you do not know your section number, write your TA’s name.) Show all work in the space immediately below each problem. Your final answer must be placed in the box provided. Problems will be graded on reasoning and intermediate steps as well as on the final answer. Be sure to include units wherever necessary, and the direction of vectors. Each problem is worth 25 points. In doing the problems, try to be neat. Check your answers to see that they have the correct dimensions (units) and are the right order of magnitude. You are allowed one 8½ x 11” sheet of notes and no other references. The exam lasts exactly 50 minutes.

(Do not write below)

SCORE:

Problem 1: FETTER

Problem 2: FEELEY

Problem 3: ALMOUGT

Problem 4: KRUGER

TOTAL: _______
1. An insulated sphere with radius \( r = 20 \text{ cm} \) has a charge of \( Q = 10 \mu \text{C} \) distributed uniformly over its surface.

   a. Calculate the electric field at a distance of 30 cm from the center of the sphere. (5 pts.)

   \[ E = \frac{kQ}{r^2} = \frac{9 \times 10^9 \times 10^{-5}}{(0.3)^2} \]

   \[ = 10^6 \text{ V/m} \]

   b. Calculate the electric potential at the center of the sphere (relative to infinity). (5 pts.)

   \[ V = \frac{kQ}{r} = \frac{9 \times 10^9 \times 10^{-5}}{0.2} \]

   \[ = 4.5 \times 10^5 \text{ kV} \]

   c. How much energy is required to remove an electron from the sphere and place it infinitely far away? (5 pts.)

   \[ U = eV = 1.6 \times 10^{-19} \times 4.5 \times 10^5 \]

   \[ = 7.2 \times 10^{-14} \text{ J} \]

   d. What is the capacitance of the sphere relative to infinity? (5 pts.)

   \[ C = \frac{Q}{V} = \frac{10^{-5}}{4.5 \times 10^5} \]

   \[ = 2.22 \times 10^{-11} \text{ PF} \]

   e. What is the total electric flux leaving the sphere? (5 pts.)

   \[ \Phi = \frac{Q}{\varepsilon_0} = \frac{10^{-5}}{8.85 \times 10^{-12}} \]

   \[ = 1.13 \times 10^6 \text{ Wb} \]
2. Two parallel conducting plates, each with area $A = 0.5 \, \text{m}^2$, are separated by a distance of $d = 1 \, \text{mm}$ and have a voltage difference of $V = 80 \, \text{volts}$.

a. Calculate the magnitude of the electric field in the space between the plates. (5 pts.)

$$E = \frac{V}{d} = \frac{80}{10^{-3}}$$

$$= 8 \times 10^4$$

$$= 80 \, \text{kV/m}$$

b. Calculate the capacitance of the plates. (5 pts.)

$$C = \frac{\varepsilon_0 A}{d} = \frac{8.85 \times 10^{-12} \times 0.5}{0.001}$$

$$= 4.425 \times 10^{-9}$$

$$= 4.425 \, \text{nF}$$

c. Calculate the charge on the positive plate. (5 pts.)

$$Q = CV = 4.425 \times 10^{-9} \times 80$$

$$= 3.54 \times 10^{-7}$$

$$= 3.54 \times 10^{-7} \, \text{C}$$

d. Calculate the energy stored in the capacitor. (5 pts.)

$$U = \frac{1}{2} CV^2 = \frac{1}{2} \times 4.425 \times 10^{-9} \times 80^2$$

$$= 1.42 \times 10^{-5}$$

$$= 14.2 \, \mu\text{J}$$

e. For how long would a current of 1 $\mu\text{A}$ have to flow to fully discharge the capacitor? (5 pts.)

$$t = \frac{Q}{I} = \frac{3.54 \times 10^{-7}}{10^{-6}}$$

$$= 3.54$$

$$= 0.354 \, \text{s}$$
3. In the circuit below, the capacitor is initially discharged when the switch is closed.

\[ E = 9 \, \text{V} \]

\[ R = 5 \, \text{M} \Omega \]

\[ C = 1 \, \mu \text{F} \]

a. What is the current in the resistor just after the switch is closed? (5 pts.)

\[ I_0 = \frac{E}{R} = \frac{9}{5 \times 10^6} \]

\[ = 1.8 \times 10^{-6} \]

\[ = 1.8 \, \text{mA} \]

b. Calculate the voltage across the capacitor 3 seconds after the switch is closed. (10 pts.)

\[ V_c = E \left(1 - e^{-\frac{t}{RC}}\right) \]

\[ = 9 \left(1 - e^{-\frac{3}{5}}\right) \]

\[ = 4.06 \, \text{V} \]

\[ 4.06 \, \text{V} \]

c. How much total energy is supplied by the battery during the time required to fully charge the capacitor? (10 pts.)

\[ U = \int E \, I \, dt \]

\[ = E \int I_0 \left(e^{-\frac{t}{RC}}\right) dt \]

\[ = E \int_0^\infty \frac{I_0}{RC} \, e^{-\frac{t}{RC}} \, dt \]

\[ = \frac{E I_0}{RC} = C E^2 \]

\[ = 10^{-6} \times 81 \]

\[ = 81 \, \mu \text{J} \]

\[ 81 \, \mu \text{J} \]
4. In the circuit below the batteries and resistors are ideal.

\[ R_1 = 3 \, \Omega \]

\[ \varepsilon_1 = 12 \, V \]

\[ \varepsilon_2 = 6 \, V \]

\[ R_2 = 6 \, \Omega \]

\[ R_3 = 3 \, \Omega \]

\[ 2 \, \Omega \]

a. Calculate the voltage across resistor \( R_1 \). (10 pts.)

\[ V_1 = (\varepsilon_1 - \varepsilon_2) \frac{R_1}{R_1 + R} \]

\[ = (12 - 6) \frac{3}{3 + 2} \]

\[ = 3.6 \, V \]

b. Calculate the power dissipated in resistor \( R_1 \). (5 pts.)

\[ P = \frac{V_1^2}{R_1} = \frac{3.6^2}{3} \]

\[ = 4.32 \, W \]

c. Calculate the current in resistor \( R_3 \). (10 pts.)

\[ I_3 = I \frac{R_2}{R_2 + R_3} \]

\[ = (\varepsilon_1 - \varepsilon_2) \frac{R_2}{R_1 + R} \frac{R_2}{R_2 + R_3} \]

\[ = \frac{12 - 6}{3 + 2} \frac{6}{6 + 3} = 0.8 \]