Exam 1

Print your name and section clearly on all five pages. (If you do not know your section number, write your TA's name.) Show all work in the space immediately below each problem. Your final answer must be placed in the box provided. Problems will be graded on reasoning and intermediate steps as well as on the final answer. Be sure to include units wherever necessary, and the direction of vectors. Each problem is worth 25 points. In doing the problems, try to be neat. Check your answers to see that they have the correct dimensions (units) and are the right order of magnitudes. You are allowed one 8½ x 11" sheet of notes and no other references. The exam lasts exactly 50 minutes.

(Do not write below)

SCORE:

Problem 1: 

Problem 2: 

Problem 3: 

Problem 4: 

TOTAL: 

1. A point charge $q_1$ is located at $x = -60$ cm and a point charge $q_2$ is located at $x = +60$ cm on the x-axis.

   a. If $q_1 = -q_2 = 7 \times 10^{-9}$ C, what is the electric field at $y = +50$ cm on the y-axis? (10 pts.)

   $$E_{y} = 0 \quad \text{by symmetry}$$

   $$E_{x} = \frac{k \cdot \frac{q}{r^2} \cdot x}{r} \quad \text{where} \quad r = \sqrt{x^2 + y^2}$$

   $$= \frac{2 \cdot k \cdot \frac{q}{r^2} \cdot x}{(x^2 + y^2)^{3/2}} = \frac{2 \cdot 9 \times 10^{-9} \cdot 7 \times 10^{-9} \cdot 0.6}{(0.36 + 0.25)^{3/2}}$$

   $$= 159 \text{ V/m (to the right)}$$

   $$159 \text{ V/m}$$

   b. If $q_1 = +q_2 = 7 \times 10^{-9}$ C, what is potential (relative to infinity) at $y = +50$ cm on the y-axis? (10 pts.)

   $$\Phi = \frac{k \cdot \frac{q}{r^2}}{r} = \frac{k \cdot q}{\sqrt{x^2 + y^2}}$$

   $$= \frac{2 \cdot 9 \times 10^{-9} \cdot 7 \times 10^{-9}}{\sqrt{0.36 + 0.25}}$$

   $$= 161 \text{ V}$$

   $$161 \text{ V}$$

   c. If $q_1 = -q_2 = 7 \times 10^{-9}$ C, what is the force between the charges? (5 pts.)

   $$F = \frac{k \cdot \frac{q_1 \cdot q_2}{r^2}}{4 \cdot x^2} = \frac{k \cdot q^2}{4 \times x^2}$$

   $$= \frac{9 \times 10^{-9} \times 4.9 \times 10^{-18}}{4 \times 0.36}$$

   $$= 3.06 \times 10^{-7} \text{ N (attractive)}$$

   $$3.06 \times 10^{-7} \text{ N}$$
2. A long, cylindrical conductor (radius = 1.0 mm) carries a charge of $4 \times 10^{-12}$ C/m and is inside a second coaxial, hollow, cylindrical conductor (inner radius = 3.0 mm, outer radius = 4.0 mm) that has a charge of $-8 \times 10^{-12}$ C/m.

a. What is the electric field 2.0 mm from the axis of the conductors? (15 pts.)

\[
\oint \vec{E} \cdot d\vec{A} = \frac{q}{\varepsilon_0} \\
E = \frac{1}{2 \pi \varepsilon_0 n} \frac{dg}{dl} \\
E = \frac{4 \times 10^{-12}}{2 \pi \times 8.85 \times 10^{-12} \times 2 \times 10^{-3}} \\
= 36.0 \text{ V/m (radially outward)}
\]

b. What is the electric field 5.0 mm from the axis of the conductors? (10 pts.)

\[
E = \frac{-4 \times 10^{-12}}{2 \pi \times 8.85 \times 10^{-12} \times 5 \times 10^{-3}} \\
= 14.4 \text{ V/m (radially inward)}
\]
3. A 20-μF capacitor charged to 2 kV and a 40-μF capacitor charged to 3 kV are suddenly connected to each other with the positive plate of each connected to the negative plate of the other.

   a. What total energy is stored in the capacitors before they are connected? (6 pts.)
   
   \[ U = \frac{1}{2} C_1 V_1^2 + \frac{1}{2} C_2 V_2^2 \]
   
   \[ = 10 \times 10^{-6} \times 4 \times 10^6 + 20 \times 10^{-6} \times 9 \times 10^6 \]
   
   \[ = 220 \text{ J} \]

   b. What is the charge on the 20-μF capacitor after the two are connected? (7 pts.)
   
   \[ q = C_1 V_1 - C_2 V_2 = 20 \times 10^{-6} \times 2 \times 10^3 - 40 \times 10^{-6} \times 3 \times 10^3 \]
   
   \[ = -80 \times 10^{-3} \text{ (total charge)} \]

   After connection, voltages are the same and charge divides in proportion to \( C \).

   \[ q_1 = q \frac{C_1}{C_1 + C_2} = - \frac{80 \times 10^{-3} \times 20}{20 + 40} = -26.7 \times 10^{-3} \text{ C} \]

   c. What is the voltage on the capacitors after they are connected? (6 pts.)
   
   \[ V = \frac{q}{C} = - \frac{80 \times 10^{-3}}{60 \times 10^{-6}} \]
   
   \[ = -1333 \text{ V} \]

   d. What is the total energy stored in the capacitors after they are connected? (6 pts.)
   
   \[ U = \frac{1}{2} C V^2 = 30 \times 10^{-6} \times (1333)^2 \]
   
   \[ = 53.3 \text{ J} \]

   missing energy is dissipated as heat and radiation
4. In the circuit below, a current of 1.4 A flows in the 4-Ω resistor.

![Circuit Diagram]

a. What is the equivalent resistance between points A and B? (10 pts.)

\[
R = 2 + \frac{16 \times 12}{16 + 12} = 8.9 \Omega
\]

b. What is the magnitude of the potential difference \(V_A - V_B\)? (10 pts.)

Voltage across 16Ω resistor is \(1.4 \times (8 + 4) = 16.8 \text{ V}\)

Current through 16Ω resistor is \(16.8/16 = 1.05 \text{ A}\)

Current through 2Ω resistor is \(1.4 + 1.05 = 2.45 \text{ A}\)

\(V_A - V_B = 2.45 \times 8.9 = 21.8 \text{ V}\)

\[21.8 \text{ V}\]

c. What total power is dissipated by the resistors? (5 pts.)

\[
P = \frac{V^2}{R} = \frac{(21.8)^2}{8.9}
\]

\[= 53.4 \text{ W}\]