3-D MHD Simulation of Oscillating Field Current Drive

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Abstract

Oscillating Field Current Drive (OFCD) is a proposed low frequency steady-state current drive technique for the Reversed Field Pinch (RFP). In OFCD toroidal and poloidal oscillating electric fields are applied with 90° phase difference to inject magnetic helicity. In the present work, the 3-D nonlinear, resistive MHD code DEBS is used to simulate OFCD in relaxed RFP plasmas. The present simulations are at high Lundquist number $S = 10^5$ and low aspect ratio $R/a=1.5$. The physics issues investigated are the response of background magnetic fluctuations to the oscillating fields, the relative contributions of the tearing mode dynamo and the oscillating fields to the current profile, and the sustainment and control of the steady-state current profile. Initial results with low amplitude oscillating fields shows the expected increase in magnetic helicity and current. Results with higher amplitude will also be presented.
Outline

- Basic models
- MHD simulations with DEBS code
- 1-D simulation
- Helicity replacement
  - Global comparison of the helicity replacement with standard RFP
  - Cycle average radial profiles and tearing fluctuation level
- OFCD helicity addition
  - Global study
  - The cycle average radial profiles
  - Magnetic energy spectrum and dynamo activities
Basic features of OFCD

• Oscillating Field Current Drive can be described by Ohm’s law model where the oscillating mean velocity $\tilde{V}$ in the mean part of $\tilde{V} \times \tilde{B}$ term is produced by the oscillating electric field.

\[
\text{ac current drive}
\]

\[(\overline{E}_|)_T + S(<\delta V \times \delta B >_|)_T + S(\tilde{V}_{00} \times \tilde{B}_{00})_T.(\tilde{b}_{00})_T = (\eta \overline{J}_|)_T
\]

\[
\downarrow
\]

\[=[\overline{E} \times \overline{B}]/B^2
\]

• The dynamo term, $<\delta V \times \delta B>$, is the surface average of total fluctuating tearing modes ($m, n \neq 0$).

$()_T$ denotes the time average over one OFCD period.
Basic features of OFCD, Cont’d

- OFCD can also be formulated by the magnetic helicity balance model.

\[ K = \int \bar{A} \cdot \bar{B} \, dv - \phi_p \phi_T \implies \frac{\partial K}{\partial t} = 2\phi_T V_T - 2\eta \int \bar{J} \cdot \bar{B} \, dv \]

injection \hspace{1cm} \text{dissipation}

- In steady state;

\[ \implies < \phi_T V_T > = < \eta \int \bar{J} \cdot \bar{B} \, dv > \]

- Helicity injection rate = \[ < \phi_{T0} V_{T0} + \frac{V_z V_\theta}{2\omega} > \]

\text{OFCD injection}

where < > denotes time average over one OFCD cycle, \( V_z \) and \( V_\theta \) are the ac driving voltages and \( \phi_T \) is the toroidal flux.
Characteristics of the MHD simulations

- 3-D resistive MHD code, DEBS is used to simulate OFCD in relaxed plasmas. In all the simulations Lundquist number $= \frac{\tau_R}{\tau_A}$, $S = 10^5$, and Aspect Ratio $R/a=1.5$.

- In DEBS code, axial and poloidal electric fields are imposed at the wall.

$$E_z = \varepsilon_z \sin(\omega t), \ E_\theta = \varepsilon_\theta \sin(\omega t + \pi/2)$$

Controlled by:
- 1- Amplitude of the ac fields $\varepsilon_z, \varepsilon_\theta$
- 2- Phasing $\delta$ between $E_z$ and $E_\theta$
- 3- Driving frequency ($\omega$)

- The oscillation frequency should be faster than the resistive diffusion time $\tau_R$ and slower than the tearing time scale $\tau_{Hybrid}$, plasma relaxation time. ($\tau_{Hybrid} \sim \sqrt{\tau_R \tau_A} \sim 1/\sqrt{S} < \tau_\omega < \tau_R$).
Characteristics of the MHD simulations, Cont’d

The present simulations can be classified as follows :

- **1-D MHD simulation**
  - 1-D OFCD simulation has been done using DEBS code (tearing modes = 0).
  - The goal is to investigate the response of the equilibrium background (m=n=0) to the oscillating fields.

- **3-D MHD simulation**
  - Helicity replacement by OFCD has been simulated when the loop voltage is turned off \( E_z(a) = 0 \), (helicity replacement).
  - To add helicity 3-D OFCD simulation has been done with loop voltage, (helicity addition).
1-D simulation shows the penetration of OFCD into the background.

- 1-D OFCD simulation without loop voltage has been done to study the penetration of oscillating fields into the plasma.
- Oscillating fields penetrate into the plasma and change the equilibrium background. As the result the current profile is changed through \(< \vec{V}_{00} \times \vec{B}_{00} >_|| \text{ term.} \)
OFCD produces skin current when the tearing modes do not exist, *(1-D simulation)*

- Final state , - Initial state

![Parallel current](image1)

![F vs θ](image2)
**OFCD replaces helicity** (helicity replacement)

To replace ohmic helicity with OFCD helicity and sustain plasma current, oscillating fields are imposed on the relaxed plasma in the absence of loop voltage ($E_z(a) = 0$).

The comparison of this simulation with standard RFP when the loop voltage is turned off shows the sustainment of plasma current due to OFCD.

$E_z = 80\sin(\omega t)$  
$E_\theta = 8\sin(\omega t + \pi/2)$  
$\tau_\omega = 1.05 \times 10^3 \tau_A$
Global quantities reach a quasi-steady state (helicity replacement)

- In both cases there is no ohmic helicity injection ($E_Z(a) = 0$).
- There is a net helicity injection into the plasma due to OFCD.
- After time $t=0.33 \tau_R$ plasma starts to get close to quasistationary state and helicity injection balance is fulfilled.
Helicity dissipation increases (helicity replacement)

\[ \eta \int \tilde{\mathbf{J}} \cdot \tilde{\mathbf{B}} \, dv \]

- Before \( t=0.33 \, \tau_R \) the mean helicity dissipation increases and contributes to the negative helicity rate. Both mean and tearing helicity dissipation rates are higher than the helicity dissipation in standard RFP.

- Steady state \( \Rightarrow \bar{\phi} \cdot \overline{V_T} = \eta \int \tilde{\mathbf{J}} \cdot \tilde{\mathbf{B}} \, dv \approx 40 \)
The dynamo term drives current in core and suppresses the current at the edge
(helicity replacement)

\[ E_{||} = \eta J_{||} - S < \tilde{V} \times \tilde{B} >_{||} , \quad \eta = (1 + 9(r/a)^2)^2 \]
\[ < \tilde{V} \times \tilde{B} > \text{ (tearing)} + < \tilde{V}_{00} \times \tilde{B}_{00} > \text{ (mean)} \]

- The dynamo term drives current in core and suppresses the current at the edge. Fluctuations amplitude is two times smaller than standard RFP fluctuations. The energy spectrum shows that.
Fluctuations amplitude is two times smaller (helicity replacement)

Fluctuation amplitude

\[ F = \frac{B_\perp}{\langle B_\perp \rangle} \]

\( \theta = \frac{\langle B_\perp \rangle}{\langle B_\perp \rangle} \)
Helicity is injected when loop voltage is present (helicity addition)

- To add more helicity oscillating fields are imposed on the relaxed plasma in the presence of loop voltage.

\[ E_z = 4 + 112 \sin(\omega t) , \ E_\theta = 11 \sin(\omega t + \pi / 2), \ \tau_\omega = 1.05 \times 10^3 \tau_A \]

The cycle average helicity when the plasma starts getting close to the saturated state is higher than preexisting ohmic helicity. The cycle average of the current is slightly less than the current for standard RFP. This can be explained by the radial profiles of each term contributing in Ohm’s law.
The reversal parameter is negative (helicity addition)

- $F - \theta$ plot shows that reversal parameter, F is negative for one OFCD cycle.
Edge fluctuations amplitude is two times larger than standard RFP (helicity addition)

- In standard RFP dynamo activities suppress the current in the core and drives current at the edge. By imposing the oscillating fields on the relaxed RFP (\( E_Z(a) = 4 \)) total \( \langle \bar{V} \times \bar{B} \rangle \) term (mean due to OFCD and tearing) is three times less suppressing close to the axis, more suppressing close to reversal.
OFCD distributes the current (helicity addition)

surface and more driving at the edge. Edge fluctuations amplitude is two times larger than standard RFP.

- $J_{||}$ decreases in the core for OFCD case since total $E_{||}$ in OFCD is less than the steady $E_{||}$ in standard RFP in the core region.

- $J_{||}$ increases locally at the edge and globally decreases with OFCD.
Other tearing modes become active (helicity addition)

The radial magnetic energy spectrum $W_{mn}(r) = 1/2 \int B_r^2 d^3r$ vs. time.

- Other tearing modes that haven’t been active in standard RFP become active with OFCD. It can be seen from the spectrum and the q profile that edge modes (external) become dominant.
Other tearing modes become active (helicity addition)
Summary

- OFCD produces skin current when the tearing modes do not exist, *(1-D simulation)*.

- **Helicity replacement**
  - 3-D OFCD simulation when loop voltage is turned off shows that dynamo *drives* current in core and suppresses the current at the edge.
  - Oscillating fields tend to decrease the fluctuations amplitude in the absence of loop voltage.

- **Helicity addition**
  - For OFCD helicity addition case with loop voltage, edge fluctuations amplitude is two times larger.
  - OFCD distributes current.
  - Other tearing modes become active with OFCD.

- Future work will be examining the present cases at higher $S$. 