BOOTSTRAP-CURRENT-DRIVEN STEADY-STATE TOKAMAK

J.C. Sprott

PLP 891

January 1983

Plasma Studies

University of Wisconsin

BOOTSTRAP-CURRENT-DRIVEN STEADY-STATE TOKAMAK

J.C. Sprott

We examime here the possibility of modifying the Levitated Octupole to operate as a tokamak with a rotational transform produced entirely by the bootstrap current. We are encouraged to consider such a possibility because of the great interest in steady-state tokamaks and because recent experiments in the Octupole have shown the existence of a bootstrap current in agreement with neoclassical predictions. We first ask whether such a configuration can exist at parameters that are likely to be accessible to us, and defer until later the practical questions of how it might be achieved.

The bootstrap current density in a large-aspect-ratio, circular tokamak is given approximately by

$$j = -\left(\frac{r}{R_o}\right)^{1/2} \frac{1}{B_p} \frac{dp}{dr} \qquad (1)$$

where r is the minor radius, R_o is the major radius, B_p is the poloidal field, and p is the plasma pressure. All units are MKS. If the poloidal field is produced entirely by the bootstrap current, and if for the sake of simplicity we assume j is constant in space, then

$$B_p = \frac{1}{2} \mu_o jr \qquad (2)$$

Substituting Eq. (2) into Eq. (1) gives an expression for $\frac{dp}{dr}$:

$$\frac{dp}{dr} = -\frac{1}{2}\mu_o j^2 \sqrt{rR_o} \tag{3}$$

The pressure p(r) can then be found by integrating Eq. (3) with the boundary condition that p(a) = 0:

$$p = \frac{1}{3}\mu_o j^2 \sqrt{a^3 R_o}\,\left[1-(r/a)^{3/2}\right] \tag{4}$$

The current j can be expressed in terms of the safety factor

$$q = \frac{2B_T}{\mu_o j R_o} \tag{5}$$

where B_T is the toroidal field, to get the following:

$$p = \frac{4B_T^2}{3\mu_o q^2}\left(\frac{a}{R_o}\right)^{3/2}\left[1-(r/a)^{3/2}\right] \tag{6}$$

As a point of interest, this implies a peak beta on axis of

$$\beta = \frac{8}{3q^2}\left(\frac{a}{R_o}\right)^{3/2} \tag{7}$$

Now if one is to maintain a pressure as given by Eq. (6) in steady-state with a loss mechanism characterized by an energy confinement time τ_E, then the required heating power is

$$P = \frac{16.9a^3B_T^2}{\mu_o q^2 \tau_E} \left(\frac{a}{R_o}\right)^{1/2} \tag{8}$$

Although there is no reason to expect a non-ohmically-heated tokamak necessarily to behave like a conventional tokamak, for lack of a better estimate of τ_E, we take Alcator scaling for the electrons

$$\tau_{Ee} = 6 \times 10^{-21}\ \bar{n}a^2 \tag{9}$$

where $\bar{n}$ is the line-averaged density, and banana-regime neoclassical scaling for the ions:

$$\tau_{Ei} = 7.2 \times 10^{17} \left(\frac{a}{R_o}\right)^{3/2} \frac{(kT_i/e)^{1/2}B_T^2}{\bar{n}q^2} \tag{10}$$

The total energy confinement time is deduced from the weighted average of the two:

$$\frac{T_e+T_i}{\tau_E} = \frac{T_e}{\tau_{Ee}} + \frac{T_i}{\tau_{Ei}} \tag{11}$$

while the ratio T_i/T_e is determined from the electron energy balance assuming electron heating is solely by coulomb collisions with the ions:

$$\frac{T_i}{T_e} = 1 + 10^{32} \frac{(kT_e/e)^{3/2}}{(\bar{n}a)^2} \tag{12}$$

If we take R_o, a, B_T, and q as adjustable parameters and P, $\bar{n}$, T_e, and T_i as unknowns, we find only three independent equations: (6), (8), and (12). We thus need an additional equation which we take to be the number of poloidal ion gyroradii across the radius of the plasma:

$$N = \frac{1733aB_T}{R_o q\sqrt{kT_i/e}} \qquad (13)$$

The quantity N thus becomes an addtional parameter, and Eq. (13) closes the set. These equations were solved numerically for the Levitated Octupole parameters (R_o=1.4 meters, a=0.35 meters), and the required power is plotted vs toroidal field for various values of N and q in Figs. 1 and 2. The curves show that there is an optimum value of toroidal field that minimizes the power required to achieve a given N and q. For fields below this minimum, neoclassical ion heat conduction dominates the loss, while above the minimum, anomalous electron losses dominate.

The calculations above assume that both the electrons and ions are in the banana regime. Since $T_i > T_e$ for plasmas sustained by ICRH, the more strigent constraint is on the electrons, where we require

$$\nu_{e*} = \frac{4.0\times10^{21} ne^4 qR_o}{(kT_e)^2} \left(\frac{R_o}{r}\right)^{3/2} < 1 \qquad (14)$$

Unfortunately, this condition can never be satisfied on axis (r=0) because there are no trapped particles there. The collisionality reaches a minimum at an intermediate radius that depends on the density and temperature profile. If we evaluate ν_{e*} at r=a/2 where we assume the density is given by $\bar{n}$, we get

$$\nu_{e*} = \frac{1.1\times10^{22}\bar{n}e^4 qR_o}{(kT_e)^2} \left(\frac{R_o}{a}\right)^{3/2} \tag{15}$$

If we take $\nu_{e*}=1$ as the boundary between the collisional and collisionless regimes, we can plot this boundary on Figs. 1-2. Taking all the constraints into account, a reasonable set of target parameters that are within the capabilities of the machine and its ICRH systems is shown in table I. Note that the density $\bar{n}$ is about a factor of ten above the Murakami density limit,

$$\bar{n} = 10^{19} B_T/R_o \tag{16}$$

but this limit presumably applies only to ohmically-heated tokamaks, and even there it can be exceeded by a considerable margin as demonstrated by Macrotor which has a B_T and R_o similar to that of the proposed device.

Similar calculations were also done for the Tokapole II device. The power required is an order of magnitude less, but it is very difficult to get plasmas significantly into the collisionless regime because of the large aspect ratio (R_o=0.5 meters, a=0.12 meters). For ISX-B, however, it ought to be possible to get q=1 and N=10 at B_T=10 kG with an ion heating power of ~3 MW at a collisionality of ν_{e*}=0.07. Thus ISX-B appears at least as attractive as the Levitated Octupole for producing a suitable steady state, and any advantage offered by the Octupole would have to lie in its ability to start up without the necessity of ohmic heating. The means for doing this will be the subject of future studies.

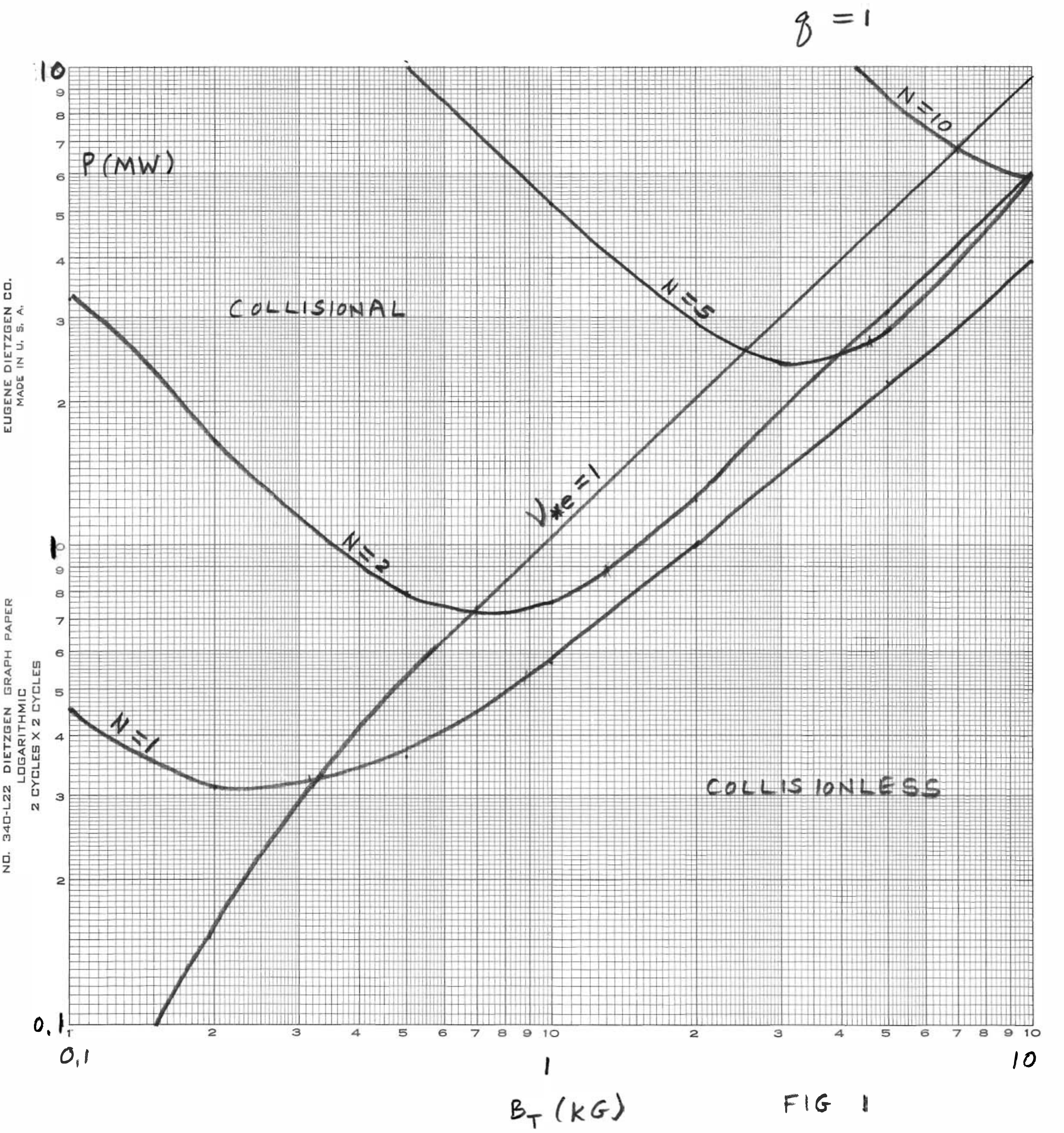
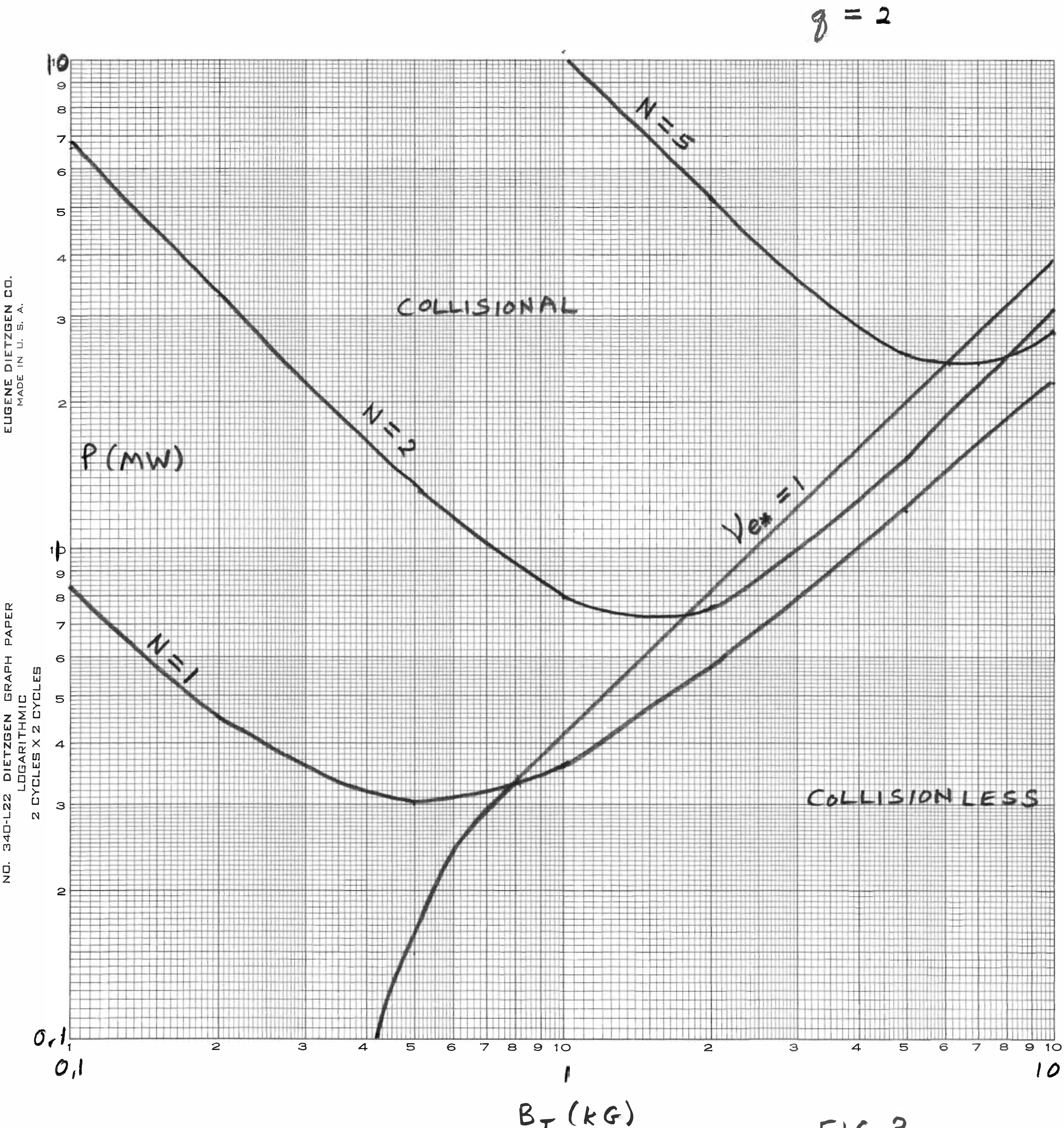

q = 1
P (MW)
COLLISIONAL
COLLISIONLESS
N = 1
N = 2
N = 5
N = 10
ν*e = 1
B_T (kG)
0.1
1
10
NO. 340-L22 DIETZGEN GRAPH PAPER
LOGARITHMIC
2 CYCLES X 2 CYCLES
EUGENE DIETZGEN CO.
MADE IN U. S. A.

FIG 1



g = 2
P (MW)
COLLISIONAL
COLLISIONLESS
N=5
N=2
N=1
Ve* =1
10
1
0,1
0,1
1
10
B_T (kG)
EUGENE DIETZGEN CO.
MADE IN U. S. A.
NO. 340-L22 DIETZGEN GRAPH PAPER
LOGARITHMIC
2 CYCLES X 2 CYCLES

FIG 2

TABLE I

R_o	(meters)	1.4
a	(meters)	0.35
B_T	(kG)	3
q		1
P	(MW)	2.4
I_P	(kA)	131
$\bar{n}$	(cm^{-3})	2.8×10^{13}
T_i	(eV)	675
T_e	(eV)	664
$\beta(0)$	(%)	33
N		5
ν_{e*}		0.53
ν_{i*}		0.37
τ_{Ee}	(msec)	21
τ_{Ei}	(msec)	7.5
τ_E	(msec)	11
τ_{ei}	(msec)	0.35
U	(kJ)	315