A MULTICOMPONENT PLASMA POWER BALANCE CALCULATION

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This note describes a computer code (MSHEX) which solves the steady state power balance equations for a multicomponent plasma of given density, in which the various species interact with one another by coulomb collisions. It was originally written to study minority species ion cyclotron resonance heating, but it also has application in calculating impurity temperatures, and in studying neutral beam heating and alpha particle slowing down. A three-component case is shown schematically below:

The input variables are the density of each specie \((n)\), the input power to each specie \((P)\), the energy confinement time for each specie \((\tau)\), and the atomic mass number \((M)\) and charge \((Z)\) of each specie. The output is the steady state temperature of each specie \((T)\).

The equations that are solved simultaneously are:

\[
P_i = \frac{1.5 n_i e T_i V}{\tau_i} - \sum_{j=1}^{3} \frac{1.5 n_j e (T_i - T_j) V}{\tau_{ij}} = 0
\]

where

\[
\tau_{ij} = 4.8 \times 10^5 \left( \frac{(M_i T_i + M_j T_j)^{1.5}}{\sqrt{M_i M_j} n_j Z_i^2 Z_j^2} \right)
\]
and \( i \) and \( j \) are indices which denote one of the three species. \( V \) is the volume of the plasma. Units are cm\(^{-3}\), eV, and sec.

The first case examined was a Tokapole II plasma with \( n = 10^{13} \text{ cm}^{-3} \) and a 1\% He\(^+\) impurity. With \( \tau = 300 \mu\text{sec} \) for each specie, the electron, ion, and helium temperatures are plotted vs ohmic heating power in Fig. 1. The results agree well with the experiment if the ohmic input power is \( \sim 400 \text{ kW} \). The helium and hydrogen ions are at essentially the same temperature, so that Doppler broadening of He II ought to give a valid ion temperature. In Fig. 2, the ohmic heating power is held constant at 400 kW, and ICRH power is added to the hydrogen. If the confinement time of each specie is constant (300 \( \mu\text{sec} \)), the hydrogen decouples from the He above \( \sim 50 \text{ eV} \), and the Doppler broadening technique is no longer suitable as an ion temperature diagnostic. The result is insensitive to the He density. If an impurity specie with a higher \( Z^2/M \) (such as \( ^{12}\text{C}^{++} \)) is used, the technique works to higher temperatures.

Fig. 3 shows what would be expected for a minority specie heating experiment in Tokapole II (95\% D, 5\% H). Such an experiment should work well, and high minority ion temperatures ought to be achieved with modest power levels. Doppler broadening of impurities would be useless as a diagnostic, however.

Fig. 4 shows the result applied to the Oak Ridge EBT-I device with 99\% D and 1\% H as a function of ECRH power. The temperatures are in reasonable agreement with the experiment if the absorbed ECRH power is the order of 10 kW. Holding the ECRH power constant at 10 kW and turning up the ICRH power gives the result shown in Fig. 5. With an ICRH power of only 1 kW, the minority ion temperature should reach 4 keV even if the confinement time remains constant (15 msec) rather than increases as neoclassical theory would predict. Such an experiment should provide an excellent test of neoclassical confinement well into the collisionless ion regime. A final case is shown in Fig. 6 where the
parameters are chosen to more nearly fit the EBT-S (scaled up EBT-I) device. The result is essentially the same except that a higher ICRH power is required.

The effect of charge exchange losses on the EBT minority specie heating experiment was assessed by adding to the two ion power balance equations a term given by

\[ p_{cx} = \frac{1.5 n_i e T_i V}{\tau_{cx}} \]

where

\[ \frac{1}{\tau_{cx}} = 4.9 \times 10^{-11} n_o \sqrt{T_i} (1 + 0.00585 T_i^{1.5}) e^{-0.0582 \sqrt{T_i}} \]

and \( n_o \) is the neutral density (assumed \( H_2 \)). The result for EBT-S with various values of \( n_o \) is shown in Fig. 7. Charge exchange is not a serious consideration for \( n_o \leq 10^{10} \text{ cm}^{-3} \).

A listing of the code is included in the appendix. It makes use of the MACC nonlinear simultaneous equation-solving routine, ZRNEQ. Convergence is rapid, and the cost is low.
** * * * **

**M** **U** **L** **I** **T** **I** **E** **R** **S** **H** **A** **R** **S** **I** **N** **G** **E** **X** **E** **C** **==** **M** **U** **L** **T** **I** **I** **O** **N** **-** **P** **R** **O** **C** **E** **S** **S** **S** **Y** **S** **T** **E** **M** **==** **V** **E** **R** **.,** **M** **A** **C** **C**

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  PROGRAM MSHEX = J.C. SPROTT - FEB 10, 1979
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3. **C** **O** **M** **O** **M** **N** **P** **(3)**, **D** **E** **N** **S** **(3)**, **T** **A** **U** **(3)**, **A** **M** **U** **(3)**, **Z** **(3)**, **V**

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9. **F** = 0, 0 E = 13

10. **D** **E** **N** **S** *(2)* = (1, 0 E = F) * Z(1) * D **E** **N** **S** *(1)* / (1, 0 E = F) * Z(2) * F * Z(3)

11. **D** **E** **N** **S** *(3)* = F * Z(1) * D **E** **N** **S** *(1)* / (1, 0 E = F) * Z(2) * F * Z(3)

12. **A** **M** **U** *(1)* = 1, 0 / 1836, 9

13. **A** **M** **U** *(2)* = 1, 0

14. **A** **M** **U** *(3)* = 4, 0

15. **V** = 5, 0 E = 8

16. **I** = 'MSHEX'

17. **D** **O** **4** **0** **I** = 1, 3

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22. **T** **A** **U** = *, **1** **P** **E** **8** *, 2*

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25. **F** **R** **M** **P** **M** **A** **T** *(1**H***, **1**)

26. **T** **2** **(E** **V**)**, **T** **3** **(E** **V**)**

27. **P** *(1)* = 4, 0 E = 5

28. **P** *(2)* = 10, 0 E = 3, 5**8** = 3**2***(**I** **P** = 1)

29. **C** **A** **L** **L** **Z** **R** **E** **F** *(T** **I** **N** **I** **T**, **A** **U** **X** **F** **C** **N***, **3**, **1**, **1** **E** = 8, 3, 1**0**0, **T** *, **0**, **L** **A**, **W** **O** **R** **K** , **I** **E** **R** , **S** **9** **0** **0**)

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32. **9** **0** **0** **C** **O** **N** **T** **I** **N** **U** **T**

33. **E** **N** **D**

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**N** **M** **A** **C** **C** **1. 17** **S** = 02/17/79 = 13: 00: 05

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7. **Z** *(D**E**N**S** *(N))/(Z*(M) * Z*(N))** 2

8. **A** **M** **U** **X** **F** **C** **N** **=** **A** **U** **X** **F** **C** **N** *(T** *(M) + T*(N))/TEQ

9. **A** **M** **U** **X** **F** **C** **N** **=** **P** *(M) = 2, 4 E = 19 * **D** **E** **N** **S** *(N) * **V** *(T*(M) / **T** **A** **U** *(M) + **A** **U** **X** **F** **C** **N**)

10. **R** **E** **T** **U** **R**

11. **E** **N** **D**
END OF COMPILATION; NO DIAGNOSTICS.

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SPECIE 2: MASS = 1.00 spec 2
SPECIE 3: MASS = 4.00 spec 3

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REQUESTS 1 $0.00
ORDS 3544 $0.00
USAGE 0.029 $0.04
IN 41 $0.03
PRINTED 1 $0.01
FILES 1 $0.06
RE SUPPORT 14 $0.12
ARGE 1 $0.03
COST $0.44

The above dollar amounts are approximate and are based on rates for overnight runs.

END OF PAGE 5
TOKAPOLE II

\[ T_e = T_i = 300 \mu \text{sec} \]

\[ n = 10^{13} \text{ cm}^{-3} \]

1% \(^4\text{He}^+\)
TOKAPOLE II

\( P_{0t} = 400 \text{ kW} \)

\( T_e = T_i = 300 \mu \text{sec} \)

\( n = 10^{13} \text{ cm}^{-3} \)

1% \( ^{4}\text{He}^+ \)

\( T_e \quad T_{\text{He}} \quad T_{\text{H}} \)

FIG 2
\[ E BT - I \]
\[ T_e = 5 \text{ msec} \]
\[ T_i = 15 \text{ msec} \]
\[ n = 2 \times 10^{12} \text{ cm}^{-3} \]
\[ 99 \% \text{ ^2D}^+ \]

FIG 4
$E_{BT} - I$

$P_{ECRH} = 10\, \text{KW}$

$n = 2 \times 10^{12}\, \text{cm}^{-3}$

$99\% \, ^2\text{D}^+$

$\tau_e = 5\, \text{msec}$

$\tau_i = 15\, \text{msec}$

$T_D$

$T_e$

$T_H$

$P_{ICRH} (\text{watts})$
EBT-S

$P_{ECRH} = 50\text{KW}$

$\tau_e = 5\text{msec}$

$\tau_i = 15\text{msec}$

$n = 5 \times 10^{12}\text{cm}^{-3}$

FIG 6
EBT-S

\[ p_{E_{CRH}} = 50 \text{ kW} \]

\[ T_e = 5 \text{ msec} \]

\[ T_i = 15 \text{ msec} \]

\[ n = 5 \times 10^{12} \text{ cm}^{-3} \]

99% \text{ } {^2}D^+