NUMERICAL CALCULATIONS OF POWER BALANCE IN THE ELMO BUMPY TORUS

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This paper describes the results obtained in a numerical solution of the steady state particle and power balance equations for the Elmo Bumpy Torus device. The program is an adaptation of program SIMULT which has been remarkably successful in predicting results for the Wisconsin Toroidal Octupoles.

The program uses a UW library subroutine (ZRNEQ) to solve a set of 4 non-linear algebraic equations consisting of a particle balance equation:

\[
\frac{dn}{dt} = \frac{dn}{dt} \text{ionization} = \frac{dn}{dt} \text{diffusion},
\]

two energy balance equations:

\[
\frac{dU_e}{dt} \text{waves} = \frac{dU_e}{dt} \text{ions} + \frac{dU_e}{dt} \text{excitation} + \frac{dU_e}{dt} \text{Bremsstrahlung} + \frac{dU_e}{dt} \text{synchrotron}
\]

+ \frac{dU_e}{dt} \text{diffusion}

\[
\frac{dU_i}{dt} \text{electrons} = \frac{dU_i}{dt} \text{charge exchange} + \frac{dU_i}{dt} \text{diffusion} + \frac{dU_i}{dt} \text{neutrals (elastic)}
\]

and a quasi-neutrality condition:

\[
\frac{dn_e}{dt} = \frac{dn_i}{dt} \text{diffusion}
\]
These four equations are solved for the four unknowns, $T_e$ (eV), $T_i$ (eV), $n_0$ (neutral density), and $\phi$ (plasma potential). Parameters in the calculation are plasma density, $n$ (cm$^{-3}$); major circumference of the torus, $L$ (cm); and minor radius of the plasma, $a$ (cm); and average magnetic field, $B$ (kG). The particle confinement time, $\tau$ (sec), is also calculated.

The ionization rate is approximated by the analytic expression:

$$\frac{dn}{dt}_{\text{ionization}} = \frac{3.71 \times 10^{-7} n_0 \sqrt{T_e} e^{-15.6/T_e}}{T_e + 15.6} \left[ \frac{T_e}{20T_e + 15.6} + \log(1.5625 + 0.1T_e) \right],$$

where $T_e$ is the electron temperature in eV. Particle losses are assumed to be governed by neo-classical diffusion:

$$\frac{dn}{dt}_{\text{diffusion}} = \frac{4.2 \times 10^{13} \sqrt{T_i} n^2 e^{\phi/T_i}}{B^2 a^2 n^2 / T_i^3 + 1.45 \times 10^{21} (T_i + 3.75\phi)^2},$$

where $\phi$ is the potential at the center of the plasma relative to the edge. (The radial electric field is assumed to be $E_r = \phi r/a$.)

The microwave heating rate is given by

$$\left. \frac{dU_e}{dt} \right|_{\text{waves}} = \frac{2 \times 10^{18} p}{a^2 L}.$$

The electron-ion energy transfer rate is given by

$$\left. \frac{dU_e}{dt} \right|_{\text{ions}} = \left. \frac{dU_i}{dt} \right|_{\text{electrons}} = \frac{2.3 \times 10^{-4} n^2 (T_e - T_i)}{T_e^{3/2}} \log \left( \frac{5.2 \times 10^{20} T_e^3}{(40 + T_e) n} \right).$$
Excitation losses are given by

\[ \frac{dU_e}{dt}_{\text{excitation}} = 29.1 e^{6.98/T_e} \frac{dn}{dt}_{\text{ionization}}. \]

Bremsstrahlung losses are given by

\[ \frac{dU_e}{dt}_{\text{Bremsstrahlung}} = 10^{-13} n^2 \sqrt{T_e}. \]

Synchrotron radiation losses are given by

\[ \frac{dU_e}{dt}_{\text{synchrotron}} = 3.87 \times 10^{-3} n B^2 T_e \left(1 + \frac{T_e}{2.04 \times 10^5}\right). \]

The energy loss through diffusion is calculated from the neo-classical model ignoring temperature gradients:

\[ \frac{dU_e}{dt}_{\text{diffusion}} = 3.5 T_e \frac{dn}{dt}_{\text{diffusion}}. \]

Similarly, the ion energy loss by diffusion is

\[ \frac{dU_i}{dt}_{\text{diffusion}} = 3.5 T_i \frac{dn}{dt}_{\text{diffusion}}. \]

Charge exchange losses are given by

\[ \frac{dU_i}{dt}_{\text{charge exchange}} = 7.32 \times 10^{-11} n_o T_i^{3/2} (1+0.00585 T_i^{3/2}) e^{-0.0582 \sqrt{T_e}}. \]

Finally, energy loss by elastic collisions with neutrals is given by

\[ \frac{dU_i}{dt}_{\text{neutrals (elastic)}} = \frac{1.88 \times 10^{-8} n_o T_i^{1.05}}{(570 + T_i^{2.5})^{0.29}}. \]
The quasineutrality condition (equal flux of electrons and ions) is given by neo-classical theory as

\[ T_e^{5/2} e^{-\phi/T_e} \left[ \frac{(10^{-9}B^2\mu^2)^2}{T_i^3} + 1370 (T_i + 3.75 \phi)^2 \right] \]

\[ = T_i^{5/2} e^{\phi/T_i} \left[ \frac{86(10^{-9}B^2\mu^2)^2}{T_e^3} + 16(T_e - 3.75\phi)^2 \right] \]

The FORTRAN listing of the program which solves this set of equations is included in the appendix along with output for a typical data run.

A number of runs were initially made with the potential \( \phi = 0 \) in order to compare with results obtained by Guest who used a more approximate form for some of the loss terms and neglected others entirely. Fig. 1 shows a case intended to be identical to a case examined by Guest, and is in remarkably good agreement with his results. A case with Bohm diffusion was also examined and is shown in Fig. 2. The experimental results appear to be more nearly consistent with neo-classical diffusion than Bohm diffusion. The effects of scaling up the microwave power density, plasma volume, and magnetic field strength are shown in Figs. 3-5 (in the \( \phi = 0 \) approximation with neo-classical diffusion).

Fig. 6 shows the result of repeating the calculation of Fig. 1 with a self-consistent plasma potential. The results are very different when the potential is included, and the striking feature is the abrupt reversal of the potential from positive to negative as the density increases above \( \sim 2 \times 10^{12} \text{ cm}^{-3} \). Such potential transitions have been observed experimentally. Fig. 7 shows the result of increasing the size of the torus by a factor of 10 (in volume), while maintaining the same microwave power density. Figs. 8 and 9 show the results of increasing the microwave power for the present
device. (Note that $V$ is the total machine value, $1.3 \times 10^6$ cm$^3$; but that the microwave power density $P/V$ is calculated using the plasma volume, $2.9 \times 10^5$ cm$^3$).

The discouraging result is that the ion temperature and particle confinement time both decrease with increasing microwave power for a constant density. However, it is known experimentally that the density increases with microwave power. Furthermore, there is experimental evidence of anomalous ion heating in certain regimes. In fact, wherever $|\psi| >> T_i$, as is the case here, we expect a rapid $\mathbf{E} \times \mathbf{B}$ rotation of the plasma with a possible enhanced energy transfer to the ions. Finally, it must be noted that the solution obtained by the computer may not be unique, and there may be other solutions which were not found.

Discussions with G. E. Guest and R. A. Dandl are gratefully acknowledged.
PROGRAM GARETH
DIMENSION XINIT(4), XFIN(4), WORK(48), DEP(31), DNP(31), TEP(31), TIP(31), PHP(31), TAP(31)
EXTERNAL AUXFCN
COMMON P, THALL, A, AL, B, DEA

DO 950, IP=1,7
P=1700.0**2.0**(IP-1)
TDENS=1
IIMAX=100
DEA=1000.0
THALL=0.025
TEA=100.0
TIA=100.0
PHI=100.0
DNFUT=0.1
AL=92.0
A=10.0
B=36.47

XINIT(1)=DNEUT
XINIT(2)=TEA
XINIT(3)=TIA
XINIT(4)=PHI
WRITE(6,400)
400 FORMAT(1H1,' I IERR DENSITY DNEUT TE')

2 TI PHI

CALL ZRNEQ(XINIT, AUXFCN, 4, 1, 0E-6, IIMAX, XFIN, 0, LA, WORK, IERR, $900)
XFIN(1)=ABS(XFIN(1))
XFIN(2)=ABS(XFIN(2))
TAU=XFIN(2)**15.6/371.0/XFIN(1)/SRT(XFIN(2))/EXP(-15.6/XFIN(2))/
2(XFIN(2)/(20.0**XFIN(2)+15.6)+ALOG(1.5625+0.1*XFIN(2)))
WRITE(6,500) IDENS, IERR, DEA, XFIN(1), XFIN(2), XFIN(3), XFIN(4), TAU
500 FORMAT(1H1,16,17,6F13.4)

DEP(IDENS)=3.0*(ALOG10(DEA)-2.0)
DNP(IDENS)=3.0*(ALOG10(ABS(XFIN(1)))-1.0)
TEP(IDENS)=3.0*(ALOG10(ABS(XFIN(2)))-1.0)
TIP(IDENS)=3.0*(ALOG10(ABS(XFIN(3)))-1.0)
PHP(IDENS)=3.0*(ALOG10(ABS(XFIN(4)))-1.0)
TAP(IDENS)=3.0*(ALOG10(ABS(TAU))-2.0)
DO 700, I=1,4
700 XINIT(I)=XFIN(I)
DEA=DEA/1.1659
KDENS=KDENS+1
IF(KDENS.LE.31) GO TO 200
900 CONTINUE

C GRAPH OUTPUT
IDENS=1
CALL GRAPH2V(DEP,IR,DP,IR,IDENS,0X6,'NONE',lZERO=D ERR SIMULAT
2ON. !, 'IDEN5-1', 'DENSITY', !, 'TEMPERATURE', !, 'I/N!')
CALL GRPH2Vv(DEP,IR,TEP,IR,IDENS,'NONE',lF!)
CALL GRPH2Vv(DEP,IR,PAP,IR,IDENS,'NONE',lV!)
CALL GRPH2Vv(DEP,IR,TAP,IR,IDENS,'NONE',lT!)
CALL GRPHND
STOP
END

OF COMPILATION: NO DIAGNOSTICS.

AUXFCN
CC. 1.14s-02/05/75-15:45:05
FUNCTION AUXFCN(X.K)
1. DIMENSION X(1)
2. COMMON P,TWALL,A,AL,B,DEA
3. DEFINE FUNCTIONS - FRT
4. D1(DENS,DNEUT,TE)=0.0*DENS*DNEUT*SOAT(TE)*EXP(-15.6/TE)*(TE/(20.
5. 20*TE+15.6)*ALOG(1.5625+1.0*TE))/((TE+15.6)
6. D2(DENS,TE)=PE4*SOAT(TM)**EXP(PHI/TIA)/R*B*A**4/TIA**3*1450.0*
7. 2*TM*TM*TM*TM**2/DENS**2)
8. PE1(P)=2.0*P/A/A/AL
9. PE2(DENS,TE,TI)=2.0*DENS**2*(TE-TI)*ALOG(5.2E11*TE**3/ABS(DENS)/
10. 20.0+TE))/TE**1.5
11. PE3(DENS,DNEUT,TE)=20.1*D1(DENS,DNEUT,TE)*EXP(6.98/(TE+0.1))
12. PE4(DENS,TE)=1.0*F=4*DENS*DENS**SOAT(TE)
13. PE5(DENS,TE)=3.87E-3*DENS**2*TE**2/DENS**2)
14. PE6(DENS,TE)=3.5*D2(DENS,TE)*(TE-TWALL)
15. PI3(DENS,DNEUT,TI)=0.0732*DENS*DNEUT*SOAT(TI)*(TI-TWALL)**1.0+0.0
16. 2*SI**2*SOAT(TI)**TI/EXP(0.058*SOAT(TI))
17. PI6(DENS,TE,TI)=3.5*D2(DENS,TE)*(TI-TWALL)
18. PI6(DENS,DNEUT,TI)=18.0*DENS*DNEUT**(TI-TWALL)**1.05/
19. (570.+TI**2.4)**.29
20. DENS=DEA
21. DNEUT=ABS(X(1))
22. TE=ABS(X(2))
23. TE=TE
24. TIA=ABS(X(3))
25. PHI=ATN1(1.0,70.0+TI))
26. GO TO (1,2,3,4,K
27. CONTINUE

DIAGNOSTIC* DEFINE PROCEDURE ARGUMENT(S) NOT USED IN DEFINITION.
AUXFCN=D1(DEA,DNEUT,TEA)-D2(DEA,TEA)
RETURN

DIAGNOSTIC* DEFINE PROCEDURE ARGUMENT(S) NOT USED IN DEFINITION.
AUXFCN=PE1(P)-PE2(DEA,TEA,TIA)-PE3(DEA,DNEUT,TEA)-PE4(DEA,TEA)-PE5
2(DEA,TEA)-PE6(DEA,TEA)
RETURN

DIAGNOSTIC* DEFINE PROCEDURE ARGUMENT(S) NOT USED IN DEFINITION.
AUXFCN=PE2(DEA,TEA,TIA)-PI3(DEA,DNEUT,TIA)-PI6(DEA,TEA,TIA)-PI8
2A,DNEUT,TIA)
RETURN
4 CONTINUE

AUXFCN = TE**2.5*EXP(-PHI/TE)*((R*A*A*DENS)**2/TI**3+1370.*(TI+3.75*2PHI)**2)-TI**2.5*EXP(PHI/TI)*((R6.0*(R*A*A*DENS)**2/TE**3+16.0*(TE-33.75*PHI)**2)

RETURN

END

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The above dollar amounts are approximate and are based on rates for standard runs.

Initiation Time: 15:44:59-FEB 5, 1975
Termination Time: 15:45:40-FEB 5, 1975
Previous Run Time: 15:28:20-FEB 5, 1975
\[ \frac{dU}{dt} \bigg|_{NC} = \frac{1}{2} T \frac{dn}{dt} \bigg|_{NC} \]

**ALL LOSSES INCLUDED**

**NEO-CLASSICAL DIFF**
$P/N = 5.76 \times 10^{-3} \text{ W/cm}^3$

**Bohm Diffusion**

$\tau = 2.5 \text{ msec}$

---

**Figure 2**
$P / \nu = 5.76 \times 10^{-3} W / \text{cc}$
$P/V = 5.76 \times 10^{-3} \text{ W/cm}^3$

NEO - CLASSICAL DIFF

WITH $\vec{E}$ FIELD

$T$ (eV), $\Phi$ (volts), $\tau$ (msec)

$n$ (cm$^{-3}$)

$T_e$, $T_i$

$-\Phi$

$\Phi$

$\tau$

$10^9$

$10^{10}$

$10^{11}$

$10^{12}$

$10^{13}$

$10^{14}$

$10^8$

$10^9$

$10^{10}$

$10^{11}$

$10^{12}$

$10^{13}$

$10^{14}$

FIG 6
\[ P/V = 5.76 \times 10^{-3} \text{ W/cm}^3 \]

\[ V = 1.3 \times 10^7 \text{ cm}^3 \]
\[ P/V = 11.5 \times 10^{-3} \text{ W/cm}^3 \]
\[ V = 1.3 \times 10^6 \text{ cm}^3 \]
$P/V = 4.3 \times 10^{-3} \text{ W/cm}^3$

$V = 1.3 \times 10^6 \text{ cm}^3$