DESIGN OPTIMATION OF OCTUPOLE ICRH COUPLING HOOP

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The design of a fifth hoop for ICRH in a toroidal octupole consists of maximizing the coupling efficiency of rf to the plasma. We look for a hoop shape and placement to maximize the induced electric field strength at the ion cyclotron resonance zone. This shape and placement will then be subject to compromises with respect to voltage standoff ability, feasibility of construction, and interference with plasma containment.

If the inductive coupling structure with the resonating capacitors form a high Q tank circuit, or in other words, if the internal impedance of the oscillator is much less than the parallel resistance of the tank circuit, we can treat the problem as though we were driving the hoop with a voltage source of voltage $E$ and frequency $\omega$.

Consider a hoop in the azimuthal direction appropriate to ICRH in a purely poloidal field. As a first approximation consider a circular cross section infinite rod near a conducting plane, with a corresponding image current below the plane (Fig. 1).

At distance $h$ above the hoop:

$$B_0 = \frac{\mu_0 I}{2\pi} \left( \frac{1}{h-d} - \frac{1}{h+d} \right) \approx \frac{\mu_0 I}{2\pi} \frac{2d}{h^2} = \frac{\mu_0 I d}{\pi h^2} \quad d<<h .$$
The inductance of the hoop is 

\[ L = \frac{\mu_0}{2\pi} \ln\left(\frac{2d}{a}\right) \]

Where \( \lambda \) is the length in the z direction. If we drive the hoop with a voltage source we have 

\[ \varepsilon = I\omega L, \quad I = \frac{\varepsilon}{\omega L} = \frac{2\pi}{\lambda\omega} \frac{2\pi}{2\omega} \left[ \ln\left(\frac{2d}{a}\right) \right] \]

and

\[ B_o = \frac{2\varepsilon d}{\lambda\omega\ln\left(\frac{2d}{a}\right)} \]

\[ B_o = \frac{\varepsilon}{\lambda\omega\ln\left(\frac{2d}{a}\right)} \frac{d}{h^2} \] for \( B_o(t) \) of the form \( B_o \sin\omega t \).

\[ VXE = -B_o \frac{\partial E_o}{\partial h} \quad \text{and} \quad E_\theta = \frac{ed}{\lambda\omega\ln\left(\frac{2d}{a}\right)} h \]

So to maximize \( E_\theta \) we must increase \( d \) and to a lesser importance \( a \).

Now consider driving two of these "hoops" in parallel with the hoops physically parallel and near. The inductance of the system is halved, the current is doubled and the field \( E \) is doubled. With this we are encouraged to consider a flat ribbon type hoop located near the wall. Again we use an infinite hoop near a conducting plane. (Fig. 2) The inductance of this hoop is

\[ L = \frac{\mu_0}{2\pi} \frac{x}{F(x)}, \quad x = \frac{\pi W}{a}, \quad F(x) = 1 + x + n(1 + x) \] where \( a \ll d \ll 2W \).

From Fig. 3 compute \( B_o \) assuming two sheet currents of density \( I/2W \) so the current flowing through \( dx \) is \( (I/2W)dx \). The x field component at height \( h \) centered above the transmission line due to the "real" current is \( d\mathbf{B}_x = \frac{\mu_0 I(h-d)dx}{4\pi W[(h-d)^2 + x^2]} \). Similarly, the contribution from the image plate is

\[ d\mathbf{B}_x = -\frac{\mu_0 I(h+d)}{4\pi W[(h+d)^2 + x^2]} \]

From handy integral tables

\[ \int_0^W \frac{dx}{a^2 + x^2} = \frac{1}{a} \arctan \frac{x}{a} \quad \left| \begin{array}{c} W \\ 0 \end{array} \right| = \frac{1}{h+d} \arctan \left( \frac{W}{h+d} \right) \]
so \[ B_{01} = 2 \int_0^W dB_1 \cdot \hat{x} = \frac{\mu_0 I}{2\pi W} \arctan\left(\frac{W}{h-d}\right) \]

\[ B_{02} = -\frac{\mu_0 I}{2\pi W} \arctan\left(\frac{W}{h+d}\right) \]

and

\[ B_0 = B_{01} + B_{02} = \frac{\mu_0 I}{2\pi W} \left[ \arctan\left(\frac{W}{h-d}\right) - \arctan\left(\frac{W}{h+d}\right) \right] = \frac{\mu_0 I d}{\pi h^2} \quad h+d \gg W. \]

Using \( L = \frac{\mu_0 d L}{2W} \) appropriate for \( x \gg l \)

\[ I = \varepsilon/\omega L, \quad B_0 = \frac{2\varepsilon W}{\pi wh^2}, \quad B_0 = \frac{2\varepsilon}{\pi h^2} = -\frac{\partial E_\theta}{\partial h} \]

and \( E_\theta = \frac{2\varepsilon W}{\pi h l} \),

so \( E_\theta \) is insensitive to \( d \) and proportional to \( W \) under the parameter ordering \( a \ll d \ll W \ll h \).

Some other considerations are the quality of electrostatic shielding of the hoop, the \( Q \) of the coil and the voltage characteristics.

Voltage standoff would argue large leakage path lengths and large radii of curvature which means a thick hoop far from the floor. Since the grounded plane is an aluminum wall of finite resistivity the higher \( Q \) will be obtained with lowest E-field at the wall which again calls for a hoop far from the wall.

The electrostatic shielding of the \( 5^{th} \) hoop is accomplished with conductive strips in the poloidal direction. The quality of electrostatic shielding rapidly deteriorates as one approaches the hoop nearer than the spacing between strips. Thus the plasma must be excluded from the region near to the hoop to maintain the shielding properties. This would require a limiter extended as far as possible above the hoop. These arguments conflict with the
requirement of minimum perturbation of the plasma. One criterion might be to require the ratio of occupied volume of public flux to private flux remain essentially unperturbed.

The analysis of the wide strip hoop can be carried further by including the first pair of image currents from the lid of the octupole and letting the computer calculate the inductance of the hoop. Referring to Fig. 4 and using the same procedure as before we find

\[ B_o = \frac{\mu_0 I}{2\pi W} [\arctan\left(\frac{W}{h-d}\right) - \arctan\left(\frac{W}{h+d}\right) + \arctan\left(\frac{W}{2z-d-h}\right) - \arctan\left(\frac{W}{2z+d-h}\right)] . \]

Write \( B_o = \alpha(h)I \) for simplicity. \( B_o = \frac{\alpha(h)E_o}{\omega L} \)

where \( L \) is the inductance per unit length of the hoop.

\[ \frac{\partial E}{\partial h} = -B_o = \omega B_o = -\frac{\alpha(h)E_o}{L} \]

where \( E_o = \varepsilon/\lambda = \) induced electric field at the hoop, and we calculate

\[ E = \frac{E_o}{L} \int_h^z \alpha(h)dh . \]

We note that at \( h=d \), \( E=E_o \) and \( E_o = \frac{E_0}{L} \int_d^z \alpha(h)dh \)

so that \( L = \int_d^z \alpha(h)dh \), defining the inductance without approximations. The calculation of \( E/E_o \) was performed on the computer to yield the curves in Figs. 5-7. Fig. 8 is a comparison of the measured electric field of the present shielded hoop and the appropriate computer calculation.

With these results in mind a new fifth hoop has been designed twice as wide and twice as far from the floor (2W=2 inches and d=1 inch) for which the E field at the resonance zone should be double that of the present hoop at the same applied voltage. Also the present hoop breaks down electrically at 3kv zero to peak, and the stainless shielding limits the Q. With the new hoop we expect applied voltages of 8kv o-p. Since the power delivered to the
plasma $\propto E^2$ we may expect a factor of 28 increase in power. Results with the present hoop are in the range 30-40eV, however the ion temperature is a sensitive function of the loss mechanisms. A computer simulation of the new hoop predicts temperatures of a few hundred ev, using a crude approximation to the charge exchange cross section. Computer simulations using refined charge exchange cross sections will be the subject of a forthcoming PLP.

REFERENCES

$E/E_0$ vs. $\frac{w}{d}$ for $\frac{d}{d} = 56$.
Fig 8

- 11 May '74 data
- Computer Calculation
- Stainless shielded
- Copper fifth hoop

$E/E_0$ vs $h/d$