COLLISIONAL CYCLOTRON

HEATING NEAR THE OCTUPOLE B = 0 AXIS

by

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March 1974

PLP 551

Plasma Studies

University of Wisconsin

Previous calculations of rf heating rates in multipoles have considered collisionless ($\nu/\omega << 1$) and collisional ($\nu/\omega_c >> 1$) cases. This note reports the results of (electron or ion) cyclotron heating calculations near the B = 0 axis in an octupole for arbitrary values of collision frequency. The results show a smooth transition from the collisionless to the collisional limit with no unexpected behavior.

As always, the starting point for the calculation is the steady state conductivity tensor which has components perpendicular and parallel to the magnetic field:

$$\sigma_\perp = \varepsilon_o \omega_p^2 \nu \left[\frac{\omega^2 + \omega_c^2 + \nu^2}{(\omega^2 - \omega^2)^2 + \nu^2(2\omega^2 + 2\omega_c^2 + \nu^2)} \right]$$

$$\sigma_\parallel = \frac{\varepsilon_o \omega_p^2 \nu}{\omega^2 + \nu^2} \quad .$$

The heating rate is given by

$$\frac{d\langle W\rangle}{dt} = \frac{\int (\sigma_\perp E_\perp^2 + \sigma_\parallel E_\parallel^2) dV}{\int n dV} \quad .$$

For an isotropic electric field

$$\frac{d\langle W\rangle}{dt} = \frac{\int (\frac{2}{3}\sigma_\perp E^2 + \frac{1}{3}\sigma_\parallel E^2) dV}{\int n dV} = \frac{e^2E^2}{m\omega} G,$$

where G is the dimensionless heating rate:

$$G = \frac{\omega \int (\frac{2}{3}\sigma_\perp + \frac{1}{3}\sigma_\parallel) E^2 dV}{\varepsilon_o \int \omega_p^2 E^2 dV} \quad .$$

If the density and electric field are homogeneous,

$$G = \frac{\omega \int \left(\frac{2}{3}\sigma_{\perp} + \frac{1}{3}\sigma_{\parallel} \right) dV}{\varepsilon_o \omega_p^2 \int dV}$$

$$= \frac{2\omega\nu}{3V} \int \frac{\omega^2 + \omega_c^2 + \nu^2}{(\omega^2 - \omega_c^2)^2 + \nu^2(2\omega^2 + 2\omega_c^2 + \nu^2)} dV + \frac{\omega\nu}{3(\omega^2 + \nu^2)} .$$

Near the octupole axis, the resonance surface is a large aspect ratio toroid of minor radius r_o, and the cyclotron frequency varies as

$$\omega_c = \omega \left(\frac{r}{r_o} \right)^3 .$$

Changing variables,

$$a = \frac{\nu}{\omega}, \qquad b = \frac{r_{max}}{r_o}, \quad \text{and} \qquad x = \frac{r}{r_o},$$

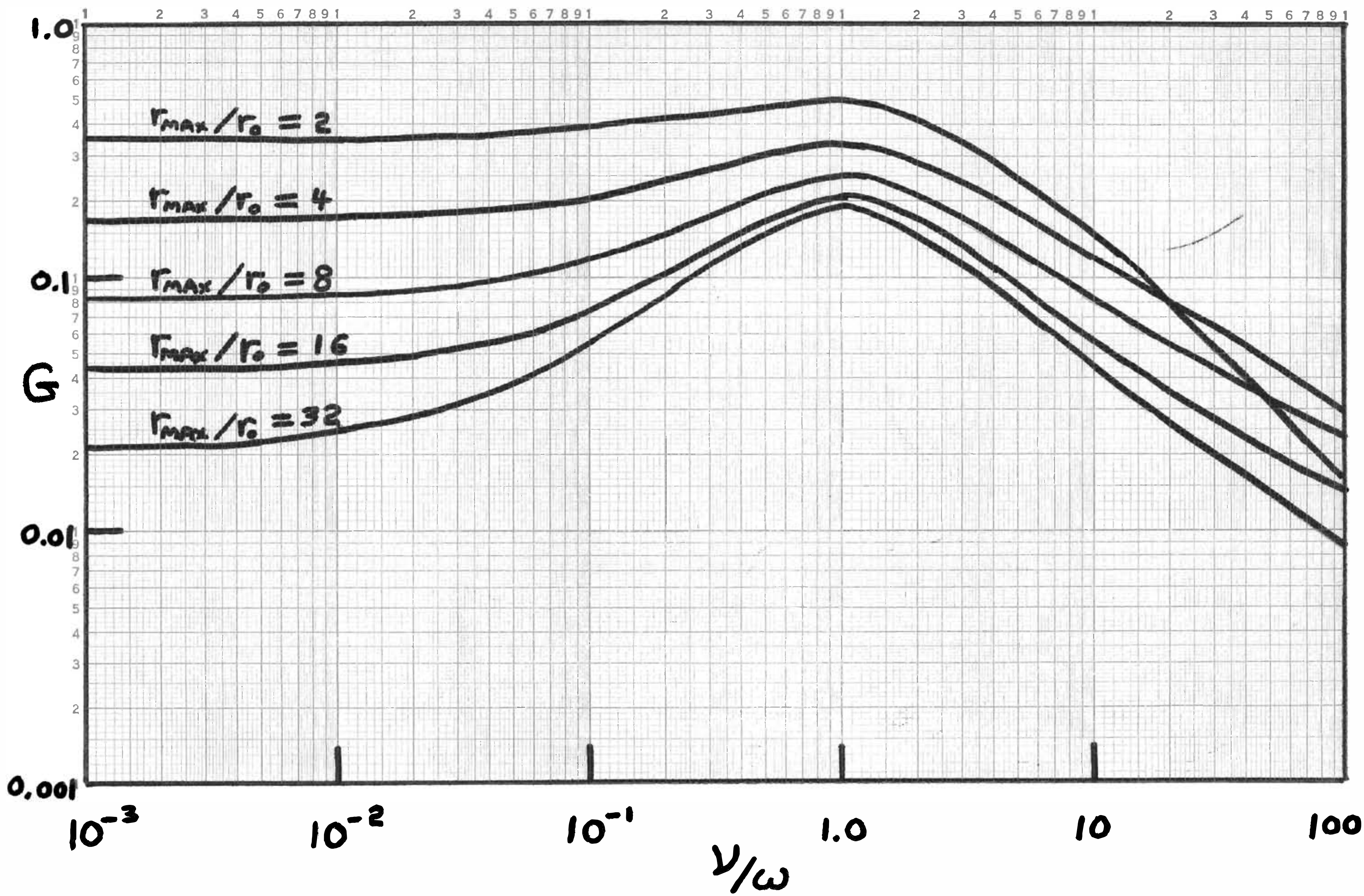
we obtain

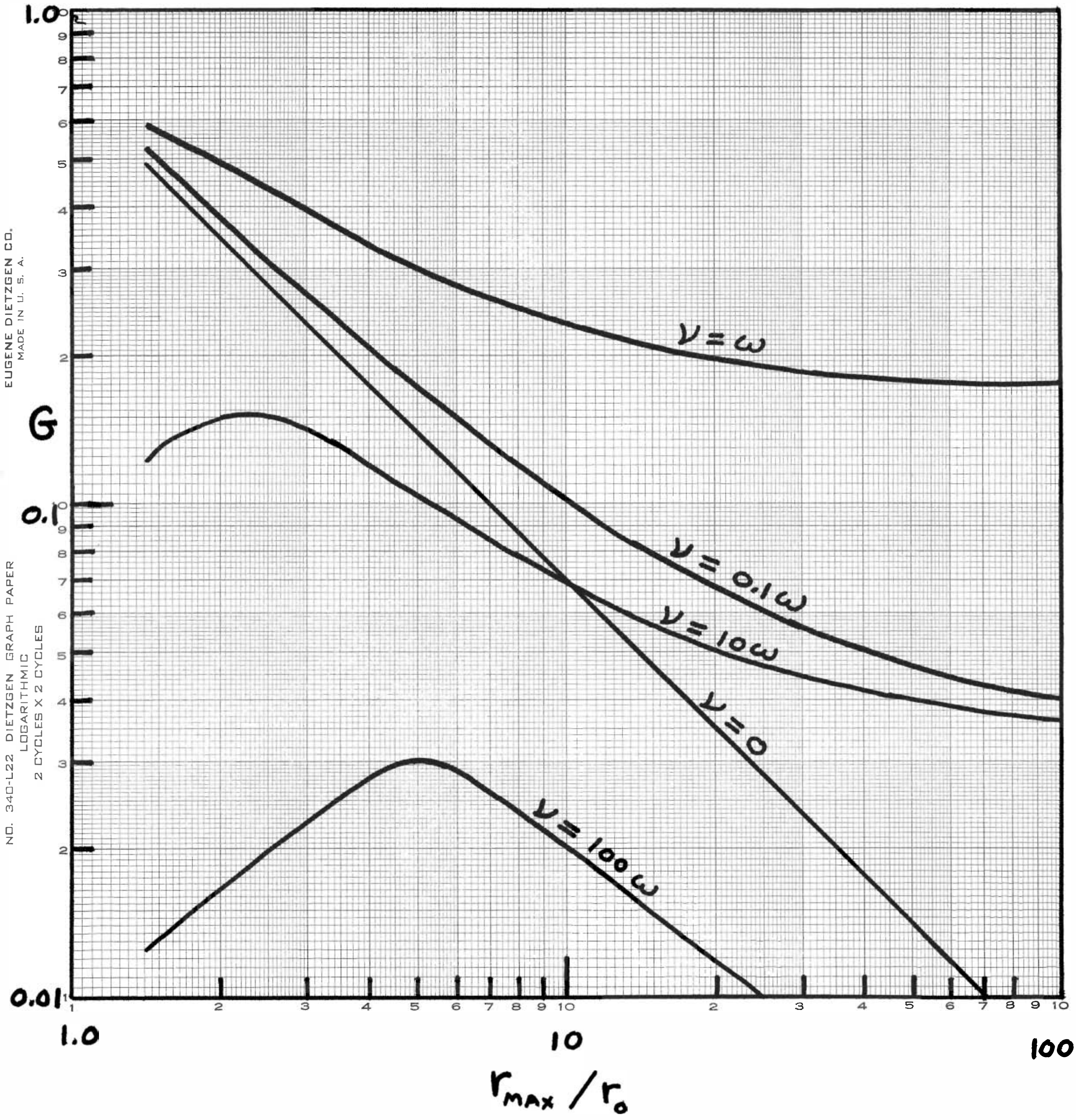
$$G = \frac{4a}{3b^2} \int_0^b \left[\frac{x^7 + a^2 x + x}{(1 - x^6)^2 + a^2(2x^6 + 2 + a^2)} + \frac{b}{4(1 + a^2)} \right] dx.$$

This integral has been evaluated numerically for various a (collision frequency) and b (resonance zone position), and the results are shown in the figures. The quantity r_{max} is given by

$$r_{max} = \sqrt{\frac{V}{2\pi^2 R_o}} = \begin{cases} 18.9 \text{ cm (small octupole)} \\ 55.8 \text{ cm (large octupole)}, \end{cases}$$

where R_o is the radius of the $B = 0$ axis and V is the volume of the plasma.


1.0
0.1
0.01
0.001
G
$r_{MAX}/r_o = 2$
$r_{MAX}/r_o = 4$
$r_{MAX}/r_o = 8$
$r_{MAX}/r_o = 16$
$r_{MAX}/r_o = 32$
10^{-3}
10^{-2}
10^{-1}
1.0
10
100
ν/ω



1.0
0.1
0.01
G
1.0
10
100
r_{MAX} / r_o
$\nu = \omega$
$\nu = 0.1\omega$
$\nu = 10\omega$
$\nu = 0$
$\nu = 100\omega$
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