ZERO-DIMENSIONAL STEADY STATE PLASMA
SIMULATION COMPUTER CODE

by

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major professor.
This note describes a computer code (ZEDTID) that calculates the spatially averaged electron (and ion) density, neutral density, electron temperature, and ion temperature in a steady state cylindrical plasma. It is a simplification of a time-dependent code (SIMJLT) described in PLP 505 and is much faster, more accurate, and quite adequate for a variety of quasi-steady state situations such as a cw microwave plasma in a toroidal octupole. It uses a UW library subroutine (ZRNEQ) that seeks a solution of \( N \) simultaneous non-linear algebraic equations in \( N \) unknowns. In this case, it is used to solve particle balance equations for electrons (or ions) and neutrals and energy balance equations for electrons and ions. The various particle and energy loss terms are a refined version of those presented in PLP 505 and will described in detail in a forthcoming PLP by Patau. The physical processes considered are listed below:

1. ionization
2. classical radial diffusion (e-i and e-n collisions)
3. obstacle losses
4. microwave heating (including finite cavity Q)
5. electron-ion energy equipartition
6. neutral collisions (excitation)
7. bremsstrahling
8. synchrotron radiation (ignoring reabsorption)
9. radial energy transport (ignoring VT)
10. ion charge exchange
11. neutral shielding (thermals and Franck-Condons)
12. finite beta
A large number of cases have been investigated, and a representa-
tive sample are described below. For each case considered, the
microwave power is varied from 1 watt to $10^6$ watts. Convergence
is obtained only if a reasonably close initial trial solution is
 supplied. This was usually done by using SIMULT with constant
field and low power to approach a steady state. Initial trial
solutions for successively higher power levels are determined by
fitting a quadratic curve to the preceding three solutions for
each unknown. In spite of this, convergence usually fails before
the power reaches $10^6$ watts. It should also be pointed out that
other solutions may exist depending on the time history of the ap-
proach to a steady state.

**Small octupole:** The terms used to describe confinement in
the small octupole are identical to those used in the refined ver-
sion of SIMULT. (In fact, the function statements can be directly
transferred from one program to the other.) The parameters used
are as follows: $T_{\text{wall}} = 0.025$ eV (room temperature), $L = 270$ cm
(major circumference of toroid), $a = 18$ cm (adjusted so $\pi a^2 L =$ total
volume), $A_o$ (obstacle area) = 90 cm$^2$ (empirically determined from
lifetime vs energy for ions and electrons), $p = 1 \times 10^{-5}$ torr (at
plasma boundary assumed $H_2$), $B = 1$ kG (crude volume average), $f =
2.45$ GHz, and $Q = 2000$ (without plasma). The results shown in Fig.
1 agree remarkably well with both experimental results and with the
peak values of the time-dependent program SIMULT. The results agree
within 1 part in $10^6$ with the steady state ($B =$ constant) infinite
time limit of SIMULT. The electron temperature stays nearly constant
at ~ 5 eV while the density increases linearly with microwave power. The computer printout for this case is included in the appendix.

**Large octupole:** For the large octupole, the parameters were changed to the following: \( L = 800 \text{ cm}, a = 50 \text{ cm}, A_o = 700 \text{ cm}^2 \) (actual geometric surface area of levators), \( p = 1 \times 10^{-6} \) torr, \( B = 1 \text{ kG}, f = 2.45 \text{ GHz}, \) and \( Q = 20,000 \). The results shown in Fig. 2 are similar to those for the small octupole and agree almost perfectly with published values (Fig. 5 of Phys. Fluids 14, 1795 (1971)). With levitation \( (A_o = 0) \), Fig. 3 shows that the electron temperature is lower and the density is higher, but this case is probably unrealistic because with classical radial diffusion as the only particle loss mechanism, the time required to reach a steady state is much longer than the duration of the magnetic field pulse. For this case, the time dependent program SIMULT would be more appropriate. One use for this program would be to try various loss terms with different parametric dependences, seeking a good fit between the density vs microwave power curve and experimental measurements. In this way some information about the nature of the anomalous losses in levitated multipoles can hopefully be obtained.

**Toroidal quadrupole:** The Wisconsin toroidal quadrupole (without ohmic heating) was investigated using the following parameters: \( T_{\text{wall}} = 0.025 \text{ eV}, L = 160 \text{ cm}, a = 6 \text{ cm}, A_o = 3 \text{ cm}^2, p = 1 \times 10^{-4} \) torr, \( B = 1 \text{ kG}, f = 3 \text{ GHz}, \) and \( Q = 500 \). The results are shown in Fig. 4. The electron temperature is nearly constant at 3 eV and
the density increases linearly with microwave power. The actual
density in the experiment is considerably below the predicted
value at 10 kW input power, and this is evidence either of anom­
alous losses (perhaps instabilities) or of the absence of a steady
state (unlikely), or of a high reflected microwave power (very
likely).

**UWFCE mirror:** The electron cyclotron heated mirror device
in B442 Engineering was studied by adding a loss cone term as de­
scribed in PLP 518. The term includes scattering of electrons
and ions on one another as well as on neutrals. Ambipolar poten­
tials are also considered. The parameters are as follows: \( T_{\text{wall}} = 0.025 \text{ eV}, L = 60 \text{ cm}, a = 6 \text{ cm (limiter radius)}, A_0 = 0, p = 1 \times 10^{-4} \text{ torr}, B = 1 \text{ kG}, f = 2.45 \text{ GHz}, \) and \( Q = 500. \) Figure 5 shows that \( T_e \)
stays nearly constant at \( \sim 30 \) eV while the density increases linearly
with power in reasonable agreement with experiment. In the experi­
ment there is also a runaway component of electrons (> 10 keV) not
covered in the calculation. These energetic electrons have only a
small effect on the particle and power balance, however.

**ELMO mirror:** The Oak Ridge ELMO mirror device was treated in
the same way as above using the following parameters: \( T_{\text{wall}} = 0.025 \text{ eV}, \)
\( L = 25 \text{ cm}, a = 10 \text{ cm}, A_0 = 0, p = 5 \times 10^{-5} \text{ torr}, B = 3 \text{ kG}, f = 10.6 \text{ GHz}, \)
and \( Q = 10,000. \) The results shown in Fig. 6 are very similar to those
in Fig. 5, but differ by as much as an order of magnitude from those
predicted by the less refined Oak Ridge program SIMULEBT (PLP 489).

**ELMO Bumpy Torus:** The Oak Ridge ELMO Bumpy Torus was studied
using the following parameters: \( T_{\text{wall}} = 0.025 \text{ eV}, a = 10 \text{ cm, L} = \)
\( 175 \text{ cm}, A_0 = 0, p = 1 \times 10^{-5} \text{ torr}, B = 5 \text{ kG}, f = 18 \text{ GHz}, \) and \( Q = 10,000. \)
A neoclassical radial diffusion term as proposed by Guest (ORNL-TM-3694) was used. Ambipolar potentials were neglected, and ions were assumed to diffuse at the same rate as electrons (as suggested by Guest). Electron-neutral collisions were added, however, and the radial energy transport corresponding to this particle diffusion (ignoring VT) was taken from Kovrizhnykh (Sov. Phys. - JETP 29, 475 (1969)). The results shown in Fig. 7 give a somewhat higher density and lower temperature than was previously calculated using SIMULEBT (PLP 489). It has not been possible to get numerical convergence for powers above 1 kW, and so the interesting collisionless regime which the experiment will hopefully reach (with powers ~ 30 kW) has not been investigated. The EBT case was also run with Bohm diffusion, and the result is shown in Fig. 8. There is not a great difference from the neoclassical case, as has been pointed out before (ORNL-TM-3694). Experimental measurements should be forthcoming.
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**PROGRAM ZET10 - SFT 2, 1973**

```
DIMENSION XIN1(4), AF1(4), XOL(4), XUL(2,4)
DIMENSION IF(31), DEL(31), TEP(31), TIP(31), SUP(31)
EXTERNAL AUXFNC
COMMON (XWALL, XALARM, RESPERSION)
```

**SPECIFY PARAMETERS**

```
IPWR=1
PFAC=1.0E+032
ILMAX IS NUMBER OF ITERATIONS
FG IS PEAK MICROWAVE POWER IN WATTS
UHA IS INITIAL DENSITY IN 10**9/CC
TWALL IS WALL TEMPERATURE IN EV
TEA IS INITIAL ELECTRON TEMPERATURE IN EV
TIM IS INITIAL TEMPERATURE IN EV
AL IS LENGTH IN CM
A IS RADIUS IN CM
AG IS ELECTRICAL ARC IN 10 CM
PRES IS NEUTRAL PRESSURE IN 10**3 TORK
DF IS FIELD AT OUTER WALL IN KOAUSS
```

**SPECIFY INITIAL CONDITIONS**

```
XIN1(1)=DEA
XIN1(2)=1E8
XIN1(3)=TIA
XIN1(4)=270.0
AL=270.0
AG=18.0
AL=90.0
PRES=1.0
DF=1.0
F=2.45
G=2000.0
```

**SPECIFY INITIAL CONDITIONS**

```
A1N(1)=DEA
A1N(2)=1E8
A1N(3)=TIA
A1N(4)=270.0
DEA=270.0
DEA=90.0
PRES=1.0
DF=1.0
F=2.45
G=2000.0
```

**SOLVE STEADY STATE EQUATIONS**

```
200 CONTINUE
PSF=0.0
CALL ZKEFMI(XIN1, POF, FCN, 2, ULE, TOL, THA, THA, TPAK, IER, 2900)
CALL ZK5EF(XIN1, ULE, 1.0E6, POF, XIN1, TIA, TIA, TIA, TIA, 2900)
SOLV=1.0E4*(XIN1(1)+XIN1(4))*(XIN1(2)+XIN1(3))
SOLV=(XIN1(1)+XIN1(4))*(XIN1(2)+XIN1(3))
SMAT=(SOLV)**0.5
IF (SMAT(10)) .LT. P11.15
PASS(IPWR)=ALOG10(P11.15)
```

**SOLVING STEADY STATE EQUATIONS**

```
PASS(IPWR)=ALOG10(P11.15)
```
DELIP(IPAR)=XLOSI1*(XFIN(1)+1.0)
TBIP(IPAR)=XLOG1(SFIN(2)+1.0)
TBIP(IPAR)=XLOG1(SFIN(3)+1.0)
TBIP(IPAR)=XLOG1(SFIN(4)+1.0)
DO 700 I=1,50
IF (IPAR .LE. 1) THEN T(1)=XFIN(1)
IF (IPAR .LE. 3) THEN T(1)=SORT(XLOG2(I)*XFIN(1)*1.0)/XLOG(1)
IF (IPAR .LE. 2) THEN XLOG(I)=XLOG(1)
700 CONTINUE
C GRAPH OUTPUT
IPAR=IPAR+1
CALL GRAPH(PPK,IPAR,IPAR,IPAR,IPAR,IPAR,OH2,'ONE',ZERO-CN TIME INDEPENDENT SIMULATION','MICROWAVE POWER','DENSTY AND TEMP','ONE')
CALL GRAPH(PPK,IPAR,IPAR,IPAR,IPAR,IPAR,ONE,'ONE')
CALL GRAPH(PPK,IPAR,IPAR,IPAR,IPAR,IPAR,ONE,'ONE')
CALL GRAPH(PPK,IPAR,IPAR,IPAR,IPAR,IPAR,ONE,'ONE')
CALL GRAPH(PPK,IPAR,IPAR,IPAR,IPAR,IPAR,ONE,'ONE')
STOP
END

COMPILED: NO · DIAGNOSTICS.
FUNCTION AUXFCN (X)*
DIMENSION ALL
COMMON *TALL, T, AL, A, R, RES, B, TR, UT
C
C DEFINE FUNCTION: - OCUPPOLE
C
C D1 IS 0/UT DUE TO IONIZATION
C D1 (DENS, DNEUT, TE) = 0.85* DENS*DNEUT*SORT (TE)*EXP (-15.* TE)/( TE + 20. )
C +1.0*25 +.1*TE))/ ( TE + 15. )
C
d2 IS 0/UT DUE TO DIFFUSION
C D2 (DENS, TE) = DENS* (.35* DENS/SORT (TE) + 0.71*DNEUT*TE)/ B / A / A
C
d3 IS 0/UT DUE TO CUST. CLE LOSSES
C D3 (DENS, TL) = 3.0* DENS**2 + (TE+TI)* ALU. ( D * D + T + 5 / A B S ( T , D E N S )
C + 20. / ( D E N S + T ) ) / T + 1.9
C
C D6 IS 0/UT DUE TO EXCITATION
C D6 (DENS, DNEUT, TE) = 2.9*41 (DENS, DNEUT, TI) * EXP ( 6.96 / ( TE + 0.1 )
C
C D4 IS 0/UT DUE TO OR_RESTATE(ND)
C D4 (DENS, TE) = 1.4* DENS + DENS*SORT (TE)
C
C D5 IS 0/UT DUE TO STR. CHAKATOR RADIATION
C D5 (DENS, TE) = 3.7* DENS* 5*5*7*8*10*15* ( 1.0+ TI / 2.0*TE )
C
C D7 IS 0/UT DUE TO THERMAL CONDUCTION
C D7 (DENS, TE) = 2.0* D3 ( DENS, TE)+ 2.0* D3 ( DENS, TE)+ D4 ( DENS )*(TE-TRALL)
C
C D8 IS 0/UT DUE TO CHARGE EXCHANGE
C D8 (DENS, DNEUT, TL) = 6.01 6* DENS*DNEUT*TL + (TI+10.0 )
C
C D9 IS 0/UT DUE TO THERMAL CONDUCTION
C D9 (DENS, TL, TI) = 2.0* D2 ( DENS, TE)+ 2.0* D3 ( DENS, TE)+ D4 ( DENS )*(TI-TRALL)
C
DECN=1(1)
DENS= DEN
TEA=X(2)
IF ( TEA + T < 20 + 1 ALL) TEA = 20 + T W ALL * ALG ( EX P ( 4.0 + TEA / T W A L L ) ) + 1.1
TE = TEA
TIA=X(3)
IF ( TI A + T < 20 + 0 ) TIA = 20 + T W ALL * ALG ( EX P ( 4.0 + TI A / T W A L L ) ) + 1.0
TI = TIA
DNEUT=X(4)
U = SORT ( DENS + DENS + U, D D E N S U + ( T E A + TIA ))
SOCL=B
DT=1.0
UO = 1.0(1,2,3,4,5)
1 CONTINUE
AUXFCN = D1 ( DEAN, DNEUT, TE ) + D2 ( DEAN, TEA ) + D3 ( DEAN, TEA ) + D4 ( DECA )/ DENS
RETURN
C CONTINUE
DECN = 1(1)
AUXFCN = D1 ( DEAN, DNEUT, TE ) + D2 ( DEAN, TEA ) + D3 ( DEAN, TEA ) + D4 ( DECA )/ DENS
RETURN
S CONTINUE.
AUXFCH=(P2+(DEA+TEA+TIA)-P13+DEA-BNUT+TIA)-P16+DEA-BEATI+TIA)/DEPS
RETURN
CONTINUE
DT1=322.0*PRES*EXP(-1.2*E-6*A+B1*(DEA+BNEUT+TEA)/BNEUT)
DFC=12.0*PRES*EXP(-6.9*E-6*A+B1*(DEA+BNEUT+TEA)/BNEUT)
AUXFCH=+BNEUT-DF1-DFC
RETURN
END

COMPIATION: NO DIAGNOSTICS.
SMALL OCTUPOLÉ
1 x 10⁻⁵ TORR
2.45 GHz
LARGE OCTUPOLE
SUPPORTED
$1 \times 10^{-6}$ TORR
2.45 GHz
LARGE OCTUPOLE
LEVITATED
$1 \times 10^{-6}$ TORR
2.45 GHz

FIG 3
FIG 4

TOROIDAL QUADRUPOLE
$1 \times 10^{-4}$ TORR
3.0 GHz

DENSITY (cm$^{-3}$)

$10^{10}$

$10^{11}$

$10^{12}$

$10^{13}$

$10^{14}$

$10^{15}$

$10^{-1}$

$10^{-2}$

$10^{-3}$

$10^{-4}$

$10^{-5}$

$10^{-6}$

POWER (WATTS)

TEMPERATURE (eV)

$10^5$

$10^4$

$10^3$

$10^2$

$10$

$1$

$0.1$
UWECE MIRROR
$R = 2$
$1 \times 10^{-4}$ TORR
2.45 GHz

FIG 5
ELMO MIRROR
R = 2
5 x 10^-5 TORR
10.6 GHz

\[ \text{DENSITY (cm}^{-3}) \]

\[ 10^{14} \]
\[ 10^{13} \]
\[ 10^{12} \]
\[ 10^{11} \]
\[ 10^{10} \]
\[ 10^{9} \]
\[ 10^{8} \]

\[ \eta_0 \]
\[ \eta_e \]

\[ T_e \]
\[ T_i \]

\[ \text{POWER (WATTS)} \]

\[ 1.0 \]
\[ 10 \]
\[ 10^2 \]
\[ 10^3 \]
\[ 10^4 \]
\[ 10^5 \]

\[ \text{TEMPERATURE (eV)} \]

\[ 10^5 \]
\[ 10^4 \]
\[ 10^3 \]
\[ 10^2 \]
\[ 10 \]
\[ 1.0 \]

FIG 6
ELMO BUMPY TORUS
NEOCCLASSICAL DIFF
$1 \times 10^{-5}$ TORR
18 GHz
ELMO BUMPY TORUS
BOHM DIFFUSION
$1 \times 10^{-5}$ TORR
18 GHz

\begin{tikzpicture}
% Draw the graph axes
\draw[->] (0,0) -- (12,0) node[right] {\text{POWER (Watts)}};
\draw[->] (0,0) -- (0,12) node[above] {\text{DENSITY (cm$^{-3}$)}};
\draw[->] (0,0) -- (0,0) node[above] {\text{TEMPERATURE (eV)}};

% Draw the curves
\draw[thick] (0,0) -- (12,10^14) node[right] {$n_e$};
\draw[thick] (0,0) -- (12,10^13) node[right] {$n_0$};
\draw[thick] (0,0) -- (12,10^10) node[right] {$T_e$};
\draw[thick] (0,0) -- (12,10^9) node[right] {$T_i$};
\end{tikzpicture}

\textbf{FIG 8}