

Proposed RF Heating Experiments  
on the Levitated Octupole

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## I. INTRODUCTION

The purpose of this PLP is three-fold: 1) to propose rf heating experiments which might be performed on the new levitated octupole, 2) to predict the results of these experiments based on theoretical calculations and scaling laws observed in similar experiments on the small octupole, and 3) to describe the steps that are being taken to prepare apparatus for these experiments.

## II. GENERAL MACHINE FEATURES

There are a number of ways in which the levitated octupole differs from the small octupole that are relevant to rf heating studies:

- 1) The cavity is larger by about a factor of 3 in linear dimensions.
- 2) The magnetic field has a slightly different shape and if run at full excitation ( $V_{\text{mult}} = 5\text{kV}$ ) will be about a factor of 2 stronger than the field in the small octupole at normal excitation ( $V_{\text{mult}} = 2\text{kV}$ ).
- 3) The hoops can be transiently levitated, eliminating obstacles that cross flux surfaces.
- 4) Pumping will be by titanium gettering rather than by oil diffusion pumps, hopefully producing somewhat lower base pressures and eliminating oil from the vacuum tank walls.
- 5) The number and relative size of ports in the tank wall will be kept to a minimum, reducing magnetic field errors and microwave leaks.

In choosing a microwave system for use on the levitated octupole, it is desirable to use a system that has already been developed for the small octupole in order to facilitate comparison of the two machines and insure rapid deployment with a minimum of development. Of the available choices (Table I of PLP 282), the X-band system seems most suitable for the following

reasons: 1) It operates at the highest power level (100 kW). 2) It has the highest frequency (9000 MHz), making possible small injection ports and providing the highest density and most uniform electric field. 3) It is equipped with a 13dB ferrite isolator, a 20dB variable attenuator, and a bi-directional coupler. 4) It has been used extensively in plasma studies on the small octupole.

Electron cyclotron heating requires that the magnetic field be operated such that electron cyclotron resonance occurs somewhere within the cavity. At 9000 MHz, the required field excitation is 23% or 1.2 kV (PLP 216). At full excitation, the resonance contour will correspond approximately to  $B_0 = 1.6$  on the small octupole. For some experiments it may be desirable to have resonance deeper in the magnetic well, and this would require the use of another, lower frequency, microwave system.

The X-band system will require use of the three channel pulser presently used with the small octupole. In order to retain ECRH capabilities on the small octupole, the electronics shop is constructing a magnetron pulser that is essentially identical to channel 1 of the 3 channel pulser and is capable of powering the present S band magnetron (2J38) or any future magnetron that requires <7.5 kV for 16-144  $\mu$ sec. By purchasing an appropriate pulse transformer, the new pulser could be used with any magnetron, including the ICRH oscillator, up to a peak input power level of 1 MW.

It should be noted that the pulse length of the X-band system is presently limited to 144  $\mu$ sec by saturation of the pulse transformer in the three channel pulser. A larger transformer would permit operation up

to the 720  $\mu$ sec limit of the pulse forming networks, provided the magnetron could dissipate the energy. Even longer pulses could be achieved by purchasing more pulse forming networks at a cost of ~\$500/msec.

In order to calculate the rf electric field in the cavity, it is necessary to estimate the cavity Q in the absence of plasma. The measured Q of the small octupole is about 4000 at 9000 MHz. (See PLP 186) There are a number of differences between the two machines that should effect the Q: 1) The Q scales like the cavity volume/surface area and hence should be three times larger for the levitated octupole. 2) Although the new swivel ports are not microwave tight, there should be fewer microwave leaks, especially around the hoop supports in the levitated octupole. 3) The vacuum tank walls should be cleaner because of the titanium gettering, although several square meters of the surface will be covered with titanium whose resistivity is ~30 times greater than aluminum.

A direct calculation based on the equation

$$\frac{1}{Q} \approx \frac{A_1 \delta_1 + A_2 \delta_2}{V},$$

where  $A_1$  and  $A_2$  are the areas of aluminum and titanium (~56m<sup>2</sup> and ~4m<sup>2</sup>) and  $\delta_1$  and  $\delta_2$  are the respective skin depths ( $8.5 \times 10^{-5}$ cm and  $4.6 \times 10^{-4}$ cm @ 9000 MHz), gives a Q of ~120,000. Taking all these facts into account we conclude that we might reasonably expect a Q of ~40,000 or 10 times larger than for the small octupole.

The vacuum electric field can be calculated from

$$\frac{1}{E^2} = \frac{P_0 Q}{\epsilon_0 \omega V},$$

giving an rms electric field of  $\sim 300$  volts/cm, or a perpendicular rms field of  $\sqrt{E_1^2} = \sqrt{\frac{2}{3} E^2} \sim 250$  volts/cm.

A Q of 40,000 gives a mode half width of  $\frac{f}{Q} \sim 225$  kHz at 9000 MHz. The average mode spacing (PLP 282) is

$$\frac{df}{dN} = \frac{1}{8\pi} \frac{\lambda^3}{V} f \sim 1.5 \text{ kHz},$$

so that the modes overlap considerably at X band. Consequently, the cavity impedance should be constant, and the problem of matching the waveguide to the cavity reduces to that of matching to free space.

In such a large cavity, one might ask whether the rf will distribute itself uniformly throughout the cavity. Crudely, the distance a wave goes before damping to 1/e is given by

$$d \sim Q\lambda \approx \frac{V\lambda}{A\delta}.$$

The cavity field is uniform provided d is much greater than a characteristic length of the cavity such as V/A, or

$$d \sim \frac{V\lambda}{A\delta} \gg \frac{V}{A} \Rightarrow \lambda \gg \delta.$$

The above condition is easily satisfied at 9000 MHz.

One of the most difficult problems is providing a suitable transition between the waveguide input and the cavity. A number of conditions must be met: 1) The wall currents producing the multipole field should be disturbed as little as possible. 2) The port should be large to reduce the electric fields and tapered to effect a more gradual impedance transformation and reduce reflections. 3) The waveguide window should be transparent to microwaves, have a low vapor pressure, and be able to

withstand plasma bombardment without giving off impurities. A slotted port made from a copper bung (see PLP 299) is presently being designed and will hopefully meet the requirements.

### III. ELECTRON CYCLOTRON HEATING EXPERIMENTS

There are two types of experiments that one might do with rf heating. One type consists of keeping the background gas pressure as low as possible and heating a gun injected plasma by rf at the electron cyclotron frequency. The other type consists of raising the pressure to produce a cold ion plasma when the rf is on.

The heating rate for the first case can be estimated from equations in PLP 282. At low densities, where the Q of the cavity is unperturbed by the plasma, the heating rate is given approximately by

$$\frac{d\bar{W}}{dt} = \frac{\pi}{2} \frac{e\bar{E}_{\perp}^2}{B_0} \left[ \frac{B_0}{V} \frac{dV}{dB} \right]_{B_0} \sim 300 \text{ eV}/\mu\text{sec}.$$

For a 144  $\mu\text{sec}$  pulse, the average electron energy should rise to  $\sim 40$  keV. This value is well above the energies obtained in the small octupole where support losses are believed to limit the electron energy. If the levators are left in place, the electron energy should reach an equilibrium at a value that can be estimated by balancing the heating and loss rates:

$$\frac{\pi}{2} \frac{e\bar{E}_{\perp}^2}{B_0} \left[ \frac{B_0}{V} \frac{dV}{dB} \right]_{B_0} \approx \frac{A}{V} \sqrt{\frac{2\bar{W}}{m}} \bar{W}.$$

Letting  $V = 8.6\text{m}^3$  and  $A = 720\text{cm}^2$  (geometrical area of levators), an equilibrium energy of  $\sim 1.5$  keV should be reached in a time of  $\sim 5\mu\text{sec}$ . This difference in electron energy should represent one of the most spectacular

effects of withdrawing the levators.

In order to measure 40 keV electron energies, any diagnostic apparatus placed in the field should have a surface area sufficiently small to ensure that the particle losses to it are negligible. The required surface area is given by

$$A \ll \frac{V}{\bar{t}} \sqrt{\frac{m}{2\bar{W}}},$$

or  $A \ll 5\text{cm}^2$  for  $\bar{W} = 40$  keV and  $t = 144\mu\text{sec}$ . Scintillator probes presently on hand should come close to satisfying this requirement, although it will be necessary to extend their calibration to higher energies. It will also probably be desirable to develop a miniature scintillator probe for use when the hoops are levitated.

It is also necessary that the background gas pressure be sufficiently low that ionization losses are negligible. Using  $p\tau_i \sim 10^{-9}\text{torr-sec}$  (valid over several decades of energy), we require  $p \ll 10^{-5}$  torr. This condition should be easily achieved with titanium gettering.

The energy calculation also assumes a plasma density sufficiently low that the  $Q$  is determined by cavity losses rather than by plasma absorption. The limiting density can be calculated (PLP 282) from

$$\omega^2 = \omega_p^2 Q \left[ \frac{B_0}{V} \frac{dV}{dB} \right]_{B_0},$$

or  $n \sim 2.5 \times 10^8\text{cm}^{-3}$ . "Fortunately" the plasma densities expected for gun injection are slightly below this value and hence are close to the maximum density that can be heated to the full predicted energy. For  $2.5 \times 10^8\text{cm}^{-3} < n < 10^{12}\text{cm}^{-3}$ , nearly total absorption of the rf is expected, so that the energy during the heating should build up according to

$$\frac{d\bar{W}}{dt} = \frac{P_o}{nV},$$

provided the levators are withdrawn. With hoops supported, the equilibrium energy for this case is given by

$$P_o \approx n \sqrt{\frac{2\bar{W}}{m}} A\bar{W}.$$

At energies of 40 keV, relativistic effects are important, and there is a question about the validity of using a cold plasma dielectric constant in the theory. Furthermore, there may be additional loss mechanisms that appear when the levators are withdrawn. For example, if we assume a loss mechanism that obeys the scaling law proposed by Meade (PLP 283), the equilibrium energy will be given by

$$\frac{\pi}{2} \frac{eE_{\perp}^2}{B_o} \left[ \frac{B_o}{V} \frac{dV}{dB} \right]_{B_o} \approx \frac{\bar{W}}{\tau_m} = \frac{c\bar{W}^2}{eBaA},$$

or about 4.2 keV. Hence, we would expect in this case only a slight improvement over the 1.5 keV expected for the unlevitated case.

To summarize the predictions, then, we expect that if a gun plasma with  $n < 2.5 \times 10^8 \text{ cm}^{-3}$  is injected and heated by a 144  $\mu\text{sec}$  pulse of 100 kW X-band microwaves at a background pressure well below  $10^{-5}$  torr in the levitated octupole, the average electron energy will rise linearly at a rate of  $\sim 300 \text{ eV}/\mu\text{sec}$ , so that, in the absence of losses, the energy will reach  $\sim 40 \text{ keV}$ . When the levators are not withdrawn, the energy should reach an equilibrium at  $\sim 1.5 \text{ keV}$  in  $\sim 5 \mu\text{sec}$ , where the heating is balanced by particle loss to the levators. When the hoops are levitated, the energy may still only reach  $\sim 4.2 \text{ keV}$  if the scaling law proposed by Meade holds. At higher densities ( $2.5 \times 10^8 < n < 10^{12} \text{ cm}^{-3}$ ), complete absorption of the

microwaves by the plasma is expected.

#### IV. COLD ION PLASMA CONFINEMENT EXPERIMENTS

The second type of experiment consists of producing a plasma by microwave breakdown at high pressure in the absence of any hot ion, gun injected plasma. The decay of the cold ion plasma produced in the afterflow would be studied. The microwave plasma has an advantage over the gun plasma in that it is density limited rather than particle limited. The gun presently in use on the small octupole where  $n \sim 10^9 \text{cm}^{-3}$  would give a density of only  $\sim 3 \times 10^7 \text{cm}^{-3}$  on the levitated octupole because of the much larger volume. However, densities approaching  $10^{11} \text{cm}^{-3}$  are obtained by X-band microwaves on the small octupole. Since the density is limited by the condition  $\omega_p < \omega$  in the resonance zones, we would expect similar densities in the levitated octupole.

The required microwave power is given by

$$P_o = \frac{nVU_i}{t}$$

where  $U_i$  is the ionization energy, and  $t$  is the duration of the microwave pulse. For a 144  $\mu\text{sec}$  pulse, a power of 14 kW is required. The 100 kW available at X-band should then be adequate to produce densities of  $10^{11} \text{cm}^{-3}$  if the background pressure is high enough. The required pressure in the absence of preionization is such that the ionization time  $\tau_i$  is about 1/10 of the pulse length, or about  $10^{-4} \text{torr}$ . It should be possible to operate the levitated octupole at this pressure by turning off the getters, bleeding in gas, and partially closing the valve between the toroid and drift tank.

Since the heating zones are near the inner hoops, a scan between the wall and an outer hoop should show the density peaked near the separatrix

in a manner similar to that expected for the gun injected plasma. The plasma should have  $kT_i < kT_e \lesssim 1$  eV, and exhibit a rapid temperature decay as a result of inelastic electron neutral collisions.

The most interesting initial test will be a comparison of lifetime with and without levitation. Ion saturation current to a probe will not give a measure of the density decay because of the rapid cooling, although it might be useful for measuring relative lifetimes, provided the decay rate of  $T_e$  is not effected by the presence of the levators. If the test is to be meaningful, the probe area should be much less than the area of the levators ( $720 \text{ cm}^2$ ). Existing probes easily satisfy this condition. Lifetime measurements with a microwave interferometer or cavity perturbation system (which should work nicely at  $n \sim 10^{11} \text{ cm}^{-3}$ ) will also be desirable as an independent test as well as for an absolute measurement of lifetime.

With hoops supported, the lifetime should be limited to a value given by

$$\tau = \frac{V}{A} \sqrt{\frac{2\pi M}{kT_e}} \sim 30 \text{ msec} \quad (@ kT_e = 1 \text{ eV})$$

Since the field pulse is only 50 msec long, it will be difficult to detect hanger losses. With the hoops levitated, the lifetime will be infinite unless other mechanisms are present. From Meade's scaling law, we calculate

$$\tau \approx \frac{6 \times 10^{-2} \text{ sec}}{kT_e \text{ (eV)}} = 60 \text{ msec} \quad (@ kT_e = 1 \text{ eV})$$

For comparison, the Bohm lifetime is  $\sim 15$  msec at  $kT_e = 1$  eV. In either case, the improvement in lifetime would not be spectacular when the hoops are levitated.

Another interesting experiment involves the use of a low power cw microwave source to produce a hot electron, cold ion plasma with a low

background gas pressure. Density buildup will occur if the ionization rate exceeds the loss rate. For losses that increase with increasing electron temperature, there is an optimum temperature at which breakdown will occur at the lowest possible pressure. This condition occurs for  $kT_e \sim U_i$  since the ionization time becomes very long for  $kT_e < U_i$ , and the decay time becomes shorter for larger  $T_e$  with no decrease in  $\tau_i$ .

For  $kT_e = U_i \approx 15$  eV, Meade's scaling law gives a decay time of  $\tau_D \approx 4$  msec. Then, to get breakdown, we require  $\tau_i < 4$  msec, or  $p > 2.5 \times 10^{-7}$  torr.

The equations that describe the breakdown are

$$\frac{d}{dt} \left( \frac{3}{2} kT_e \right) + \frac{\frac{3}{2} kT_e + U_i}{\tau_i} + \frac{\frac{3}{2} kT_e}{\tau_D} = \frac{\pi e E^2}{2B_0} \left[ \frac{B_0}{V} \frac{dV}{dB} \right]_{B_0}$$

and

$$\frac{dn}{dt} = n \left( \frac{1}{\tau_i} - \frac{1}{\tau_D} \right)$$

where  $\tau_i$  can be approximated by

$$\tau_i \approx \frac{10^{-9} \text{sec}}{p(\text{torr})} (1 + U_i/kT_e) e^{U_i/kT_e}$$

and  $\tau_D(kT_e)$  is the decay time for whatever process dominates the particle loss.

In order to keep the equilibrium temperature at  $kT_e = U_i$ , we require

$$\frac{4U_i}{\tau_i} = \frac{\pi e E^2}{2B_0} \left[ \frac{B_0}{V} \frac{dV}{dB} \right]_{B_0},$$

or  $E \sim 1.75$  volt/cm for  $U_i = 15$  eV,  $\tau_i = 4$  msec, and  $B_0 = 3200$  gauss (X-band heating). For  $n < 2.5 \times 10^8 \text{cm}^{-3}$ , the optimum microwave power is only 5 watts. An X-band cw microwave source at this power level should be

relatively easy to develop, especially since it need be turned on only for a fraction of a second every few minutes. A 100 watt X-band cw source is in final stages of construction, and Meade has purchased several 3 kW cw klystrons for heating in his steady state device.

In conclusion, it should be a simple matter to produce a plasma with  $n \sim 10^{11} \text{cm}^{-3}$  and  $kT_i < kT_e \lesssim 1 \text{ eV}$ , by heating with a 144  $\mu\text{sec}$  pulse of 100 kW, X-band microwaves with  $p \sim 10^{-4}$  torr. The lifetime should be limited to  $\sim 30$  msec with levators in place and should increase to  $\sim 60$  msec when the hoops are levitated if Meade's scaling law is valid. A hotter ( $kT_e \sim 15 \text{ eV}$ ), more tenuous ( $\sim 2.5 \times 10^8 \text{cm}^{-3}$ ) plasma should be produced by a low power (5 watts), X-band, cw, microwave source at pressures as low as  $2.5 \times 10^{-7}$  torr.

#### V. ION CYCLOTRON HEATING

It is perhaps premature to discuss ion cyclotron heating in the levitated octupole since heating has yet to be conclusively demonstrated in the small octupole. However, several important differences between the two devices should be noted. The three phase drive system that produces rf currents in the hoops could be used on the inner hoops of the levitated octupole if the levators are left inserted. However, the outer hoops have five levators, and so a symmetric feed system would require five phase rf (which could be produced without additional complexity). The wires leading to the levators would not be suitable for the kilovolt potentials at several hundred amps required for heating, however. Furthermore, such a drive system would be useless when the hoops are levitated.

Assuming we are not permitted to put coils or other obstacles into the plasma, the only way left to generate rf electric fields in the toroid

would be to impress a voltage across some electrical gap in the vacuum tank wall. The best such place would be at the " $B_0$  gap" where the tank lid meets the outer wall. Unfortunately, the initial design of the machine does not include an insulated gap at this point, and so that experiment will have to be deferred to the future.

The only other gap is the synchrotron gap where the low frequency (10 Hz) magnetic field lines enter the machine. Unfortunately, driving this gap would be very difficult because rf flux would be induced in the transformer core, which should be very lossy at the frequencies required for ion cyclotron heating. Furthermore, the transformer primary is connected through a low inductance transmission line to a large (.048 farad) capacitor bank, which would shunt out the rf. Hence, it would be necessary to develop the rf voltage across the small leakage inductance between the primary and continuity windings of the transformer. The most effective way to accomplish this would be to place enough high voltage mica capacitors across the gap to form a resonant circuit with the leakage inductance at the rf frequency. The voltage that could be produced would then be determined by the rf power and the Q of the circuit:

$$\overline{V^2} = \frac{P_0 Q}{\omega C} = P_0 Q \omega L_L.$$

In order to determine the feasibility of this experiment, we will calculate the voltage that would be required across the synchrotron gap in order to produce significant ion heating. Suppose we require  $\Delta kT_i = 30$  eV or .1 eV/ $\mu$ sec for a 300  $\mu$ sec rf pulse. The required electric field is given by (PLP 282):

$$\frac{d\overline{W}_i}{dt} = \frac{\pi e \overline{E}_\perp^2}{2B_0} \left[ \frac{B_0}{V} \frac{dV}{dB} \right]_{B_0}.$$

For  $B_0 = 700$  gauss (field for ion cyclotron resonance for a proton at  $f = 1$  MHz), the required perpendicular electric field is  $\sim 2$  volts/cm. The required gap voltage is then given by

$$\sqrt{V^2} \frac{\psi_{\text{total}}}{\psi_{\text{private}}} 2\pi R_0 E_{\perp} \sim 2500 \text{ volts (rms)}.$$

With available apparatus (100 kW at 1 MHz), it is doubtful that a resonance circuit with sufficiently high Q could be constructed. Also, the gap may not withstand the 3500 volts peak required to produce to desired heating.

In summary, the prospects for ion cyclotron heating seem less promising for the levitated octupole than for the small octupole, at least until a  $B_{\theta}$  gap is added to the machine. Tests currently in progress on the small octupole will hopefully shed more light on this rather complicated problem.