A Comparison of ECRH in a Toroidal Octupole and a Magnetic Mirror

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In PLP 186 it was shown that for a low density plasma confined in a closed conducting cavity filled uniformly and isotropically with microwave radiation that the microwave power absorbed by the plasma is given by

$$P = \frac{\pi}{2} \; P_o Q \frac{\omega_p^{\;2}}{\omega^2} \left(\frac{B}{V_o} \; \frac{dV}{dB}\right) \qquad (1)$$

where P_o is the input power, ω_p is the electron plasma frequency, ω is the microwave frequency, B is the magnetic field at which electron cyclotron resonance occurs, and V_o is the volume of the cavity. The volume between adjacent constant-B surfaces (dV/dB) is evaluated at the resonant B surface. The quantity $(B/V_o)(dV/dB)$ for the Wisconsin toroidal octupole is plotted in Fig. 5 of PLP 186. It has a value of ~0.2 over most of the useful volume of the octupole.

It is of interest to estimate the quantity $(B/V_o)(dV/dB)$ for a typical magnetic mirror field. If we neglect any radial dependence, the magnetic field of a mirror can be represented by

$$B = B_o(1 - \alpha \cos \frac{\pi z}{z_o}) \qquad (2)$$

where α is a constant related to the mirror ratio R by

$$\alpha = \frac{R - 1}{R + 1}.$$

The volume between adjacent B surfaces is

$$\frac{dV}{dB} = \pi r^2 \left(\frac{dB}{dz}\right)^{-1} = \frac{r^2 z_o}{\alpha B_o \sin \frac{\pi z}{z_o}} .$$

If we assume that the ends of the vacuum cavity are at $z = \pm z_o$ we obtain

$$\frac{B}{V_o} \frac{dV}{dB} = \frac{1 - \alpha \cos \frac{\pi z}{z_o}}{\alpha \pi \sin \frac{\pi z}{z_o}} . \tag{3}$$

Equation (3) is plotted vs. z/z_o for various values of mirror ratio in Fig. 1. The absorption is greatest for small mirror ratios, and goes to infinity for perfectly uniform fields. Note that $(B/V_o)(dV/dB)$ tends to infinity at $z = 0$ and $z = z_o$ as would be expected since $\nabla B = 0$ there. These singularities are unphysical for two reasons:

1) In a real mirror, there would be a radial field gradient.

2) The derivation breaks down when (dB/dV) goes to zero, and second order terms $(V)(d^2B/dV^2)$ must be considered.

Nevertheless, it is clear, even for mirror ratios as high as 10 to 1, that the absorption is considerably higher than for the toroidal octupole case where $(B/V_o)(dV/dB)$ is the order of 0.2. This difference can be understood intuitively since most of the volume in the toroidal octupole is "wasted" with space in which the magnetic field is too weak to give electron cyclotron resonance.

Equation (1) also states that the power absorbed is proportional to the Q of the cavity. Since the Q of a cavity is given approximately by

$$Q \cong \frac{V_o}{2\pi A \delta} \tag{4}$$

where A is the surface area and δ is the skin depth, it is possible to compare the Q of the toroidal octupole with that of a typical magnetic mirror. It was stated in PLP 186 that eq. (4) predicts a Q of about 10,500 at 9 GHz for the toroidal octupole. Consider a magnetic mirror enclosed in a cylindrical vacuum vessel with a diameter equal to its length ($r = z_o$). The Q of such a device would be given by $r/6\pi\delta$. If the cavity had the same volume as the toroidal octupole ($V_o = 3 \times 10^5 cm^3$), and if it were made of aluminum, its Q would be 21,000 or twice that of the toroidal octupole. Furthermore, since the mode pattern is more easily controlled in a simple cylinder, it is presumably possible to choose a particular mode which has a Q considerably higher than this typical value.

The derivation of eq. (1) assumed that the microwave electric field is randomly oriented with respect to the magnetic field. In a toroidal octupole with a complicated magnetic field and mode pattern, this is probably an excellent assumption. With a magnetic mirror, however, it is possible to propagate circularly polarized waves along the mirror axis in such a way as to maximize the absorption. In this way, a factor of two can be gained over the result of eq. (1).

The preceding discussion assumed that the plasma density was sufficiently low that the plasma caused a negligible increase in the Q of the cavity, or, mathematically,

$$Q\left(\frac{B}{V_o} \frac{dV}{dB}\right) \omega_p^2 << \omega^2. \qquad (5)$$

For high densities, not satisfying this condition, we expect the power absorbed by the plasma to be the same for both field configurations, viz., the total input power. Note, however, that the density at which total absorption occurs is considerably lower for the mirror than for the toroidal octupole since $Q(B/V_o)(dV/dB)$ is much larger for the mirror.

In PLP 142, it was shown that as a particle in a non-uniform magnetic field gains energy, its gyroradius increases until its gyrational frequency departs from the microwave frequency sufficiently to prevent further absorption of energy. The final energy reached by the particle is given by

$$W = 0.29\ m\left[a\omega\left(\frac{B}{\nabla B}\right)^2\right]^{2/3} \qquad (6)$$

where a is the average acceleration,

$$a = \frac{e}{m}\ \overline{E} = \frac{e}{m}\sqrt{\frac{4\pi P_o Q'}{V_o}}\ . \qquad (7)$$

For a given power density P_o/V_o, microwave frequency ω, and cavity Q, the energy obtainable in various magnetic field configurations can be compared by comparing the respective field uniformities $(B/\nabla B)$. The Q in eq. (7) is primed to indicate that it is evaluated in the presence of plasma.

A toroidal octupole has the interesting property that ∇B is greater than zero everywhere except on the minor axis, and that the value of $(B/\nabla B$ never exceeds about 10 cm at any place in the machine that is useful for cyclotron resonance heating. The mirror, on the other hand, has

$\nabla B = 0$ at three places on the axis - at the center and at the two throats. For $\nabla B = 0$, eq. (7) must be replaced by a similar expression involving the next higher term in the expansion:

$$E = 0.73\ m[a\omega \left(\frac{B}{\nabla^2 B}\right)\ ]^{2/3} \qquad (8)$$

Since $(B/\nabla^2 B)$ in a mirror is usually much greater than $(B/\nabla B)^2 \lesssim 100\ cm^2$ in the toroidal octupole, especially at the throats of a mirror with a small mirror ratio, the energy obtainable by cyclotron resonance heating is much greater for a mirror.

Mirrors are frequently operated in such a way as to give a resonance zone which intersects the axis somewhere between the center and the throats on the assumption that particles will mirror back and forth between resonance zones, picking up energy at each turning point. This assumption relies on the fact that the electrons are adiabatic, i.e., that $W_{\perp}/B$ = const. In fact, for most mirrors, this is a good assumption up to quite high energies. For the toroidal octupole, however, particles heated near the separatrix pass through a region in which B is very small and hence it is unlikely that they will remain adiabatic for more than a few reflections. Hence, the turning point drifts away from the resonance region at much lower energies in the octupole than in a typical mirror. Heating away from the separatrix would avoid this problem, but this produces a density dsitribution which is undesirable.

In summary, it has been shown that electron cyclotron resonance heating is less effective in a toroidal octupole than in a magnetic mirror for the following reasons:

1) The fractional resonance volume $(B/V_o)(dV/dB)$ is smaller.

2) The cavity Q is smaller.

3) The electric field distribution is more difficult to control.

4) The field gradient ∇B is larger.

5) Particles are more non-adiabatic.

Note, however, that since the power absorbed by the plasma (eq. (1)) and the maximum energy obtainable (eq. (6)) depend on the power density (P_o/V_o), that it is always possible to make a toroidal octupole perform as well as a mirror by simply increasing the power. However, it is clear that while a toroidal octupole may be an excellent containment device, it is far less amenable to electron cyclotron resonance heating than is a magnetic mirror.

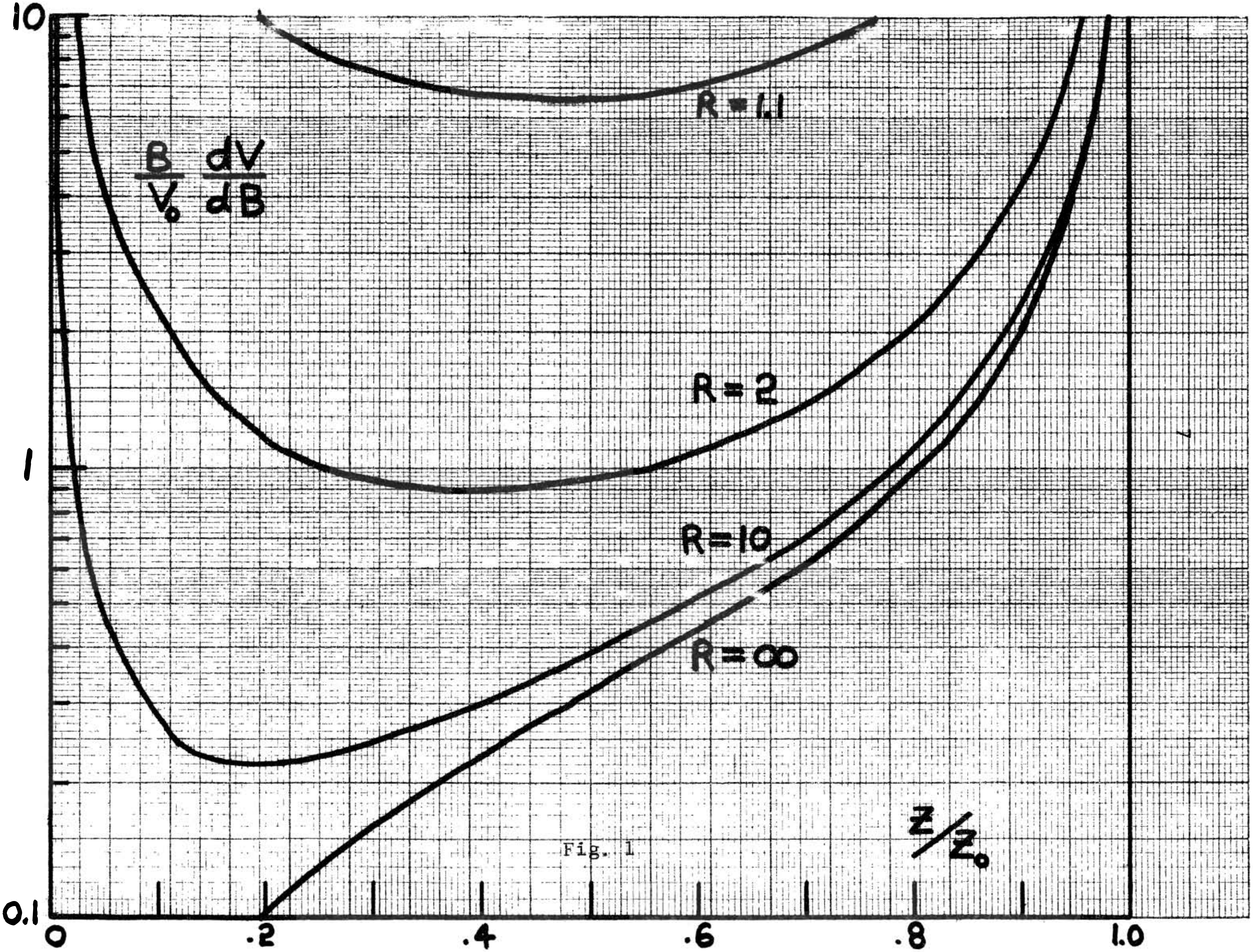

Fig. 1