Non-axisymmetric perturbations of the vacuum magnetic surfaces in a RFP

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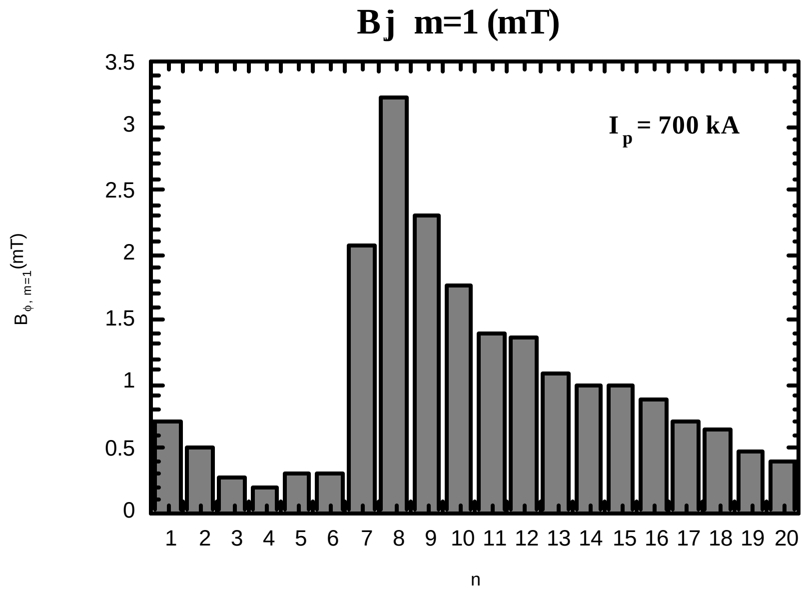
Outline

- Locked-mode and perturbation of the vacuum magnetic surfaces
- Reconstruction of the geometrical perturbation starting from magnetic probes arrays
- General method
- Approximation adopted in RFX
- Amplitude of the perturbation and scaling with the plasma current

Dynamo modes m=0,1

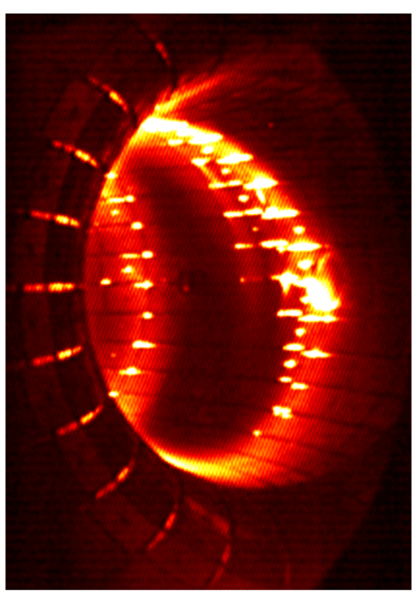


Locked mode

Phase-locking and wall-locking → localized helical distortion of the plasma surface

Severe plasma wall-interaction (PWI) phenomena.

Reconstruction of the m=0,1 magnetic surfaces perturbation



GENERAL METHOD

Approximate analytical approach: the poloidal and toroidal harmonics of the magnetic quantities are treated as perturbations.

Elementary toroidal coordinate sisyem (r, θ, φ)

Flux coordinate system (ψ, θ, φ).

The *m=0,1* distortion of a magnetic surface identified by ψ :

$$r(\psi,\theta,\varphi) \cong r_0(\psi) + \delta(\psi,\varphi) + \Delta H(\psi,\varphi)\cos\theta + \Delta V(\psi,\varphi)\sin\theta$$

The **contravariant representation** of the magnetic field is:

$$\mathbf{B} = \frac{1}{2\pi}\nabla\Psi_T \times \nabla\theta - \frac{1}{2\pi}\nabla\Psi_P \times \nabla\varphi + \nabla\psi \times \nabla\tilde{v}(\psi,\theta,\varphi) \ .$$

$$B_{r,m=0}(r,\varphi) \cong \frac{B_{\varphi 0}}{R_0}\frac{\partial\delta(r,\varphi)}{\partial\varphi}$$

$$\langle B_r \cos\theta\rangle(r,\varphi) \cong \frac{B_{\theta 0}}{r}\Delta V(r,\varphi) + \frac{B_{\varphi 0}}{R_0}\frac{\partial\Delta H(r,\varphi)}{\partial\varphi}$$

$$\langle B_r \sin\theta\rangle(r,\varphi) \cong -\frac{B_{\theta 0}}{r}\Delta H(r,\varphi) + \frac{B_{\varphi 0}}{R_0}\frac{\partial\Delta V(r,\varphi)}{\partial\varphi} \ .$$

The radial field is the key information to obtain the non-axisymmetric shape

Vacuum region between plasma and the first conductor:

$$\nabla \mathrm{x}\mathbf{B}=0 \quad \rightarrow \quad \mathbf{B}=\nabla\chi$$

$$\nabla\cdot\mathbf{B}=0 \quad \rightarrow \quad \nabla^2\chi=0$$

Perturbative solution obtained linearizing in the harmonics and in the toroidal factor r/R_0

The **boundary conditions** can be provided by the magnetic field measurements at a given radius:

- 4 toroidal arrays of radial field probes
- 4 toroidal arrays of poloidal or toroidal field probes

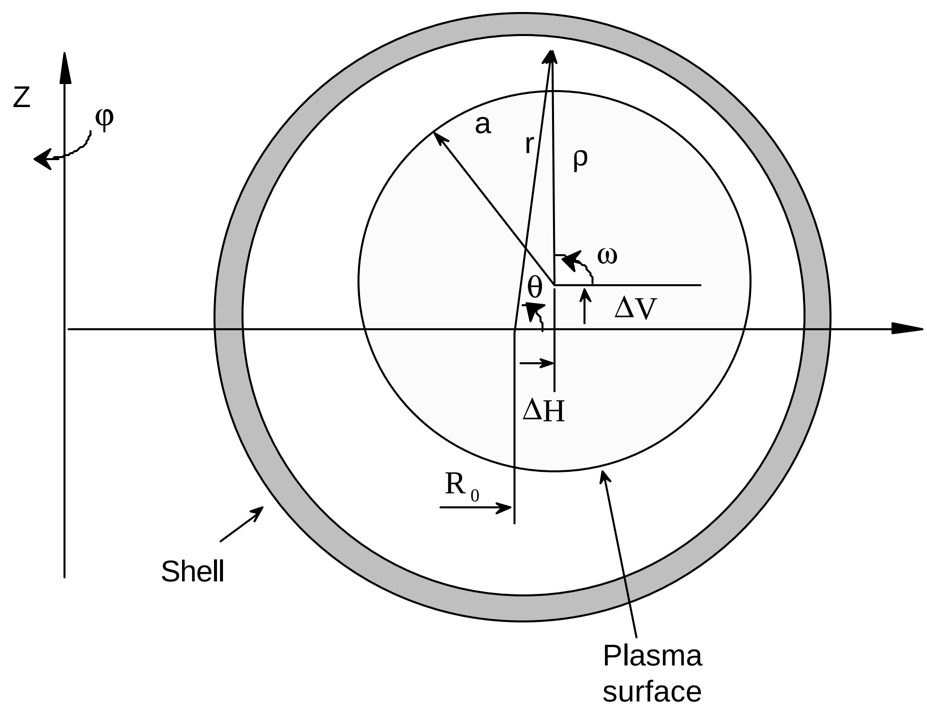
Approximate analytical expression for the m=1 vacuum magnetic field

It is interesting to solve the Laplace equation imposing as boundary conditions the relations which must be satisfied by the magnetic field at the plasma surface r = a.

From this approach approximate formulas which express the vacuum field in terms of the plasma surface shift ΔH(a), ΔV(a) can be derived.

This entails the adoption of a generalized <u>not-orthogonal</u> toroidal coordinate system $u^i = (r, w, j)$ centred with respect to the plasma surface.

$$\rho \cong r - \Delta H(a,\varphi)\cos\theta - \Delta V(a,\varphi)\sin\theta \qquad \omega \cong \theta + \frac{\Delta H(a,\varphi)}{r}\sin\theta - \frac{\Delta V(a,\varphi)}{r}\cos\theta$$



Boundary conditions

Axisymmetric case:

$$B^{\rho}(a,\omega) = 0\,; \quad B^{\omega}(a,\omega) = \frac{\mu_0 I}{2\pi a}\left(1 + \frac{a}{R_0}\Lambda_s(a)\cos\omega\right)$$

(Shafranov V.D. 1966 *Rev. Plasma Phys.* **2** 103.)

Non-axisymmetric case

$$\left\langle B^{\rho}\cos\omega\right\rangle\Big|_a = \left\langle B^{\rho}\sin\omega\right\rangle\Big|_a = 0$$

$$B^{\omega}\Big|_a = \frac{\mu_0 I}{2\pi a}\left(1 + \lambda_V(a,\varphi)\sin\omega + \frac{a}{R_0}\Lambda(a,\varphi)\cos\omega\right)$$

$$\frac{a}{R_0}\Lambda(a,\varphi) = \frac{a}{R_0}\Lambda_S(a) + \lambda_H(a,\varphi).$$

The perturbed **m=**1 vacuum magnetic field can be expressed by **generalized-Shafranov expressions**:

$$B_\theta(r,\theta,\varphi) = \frac{1}{r}\frac{\partial\chi}{\partial\theta} \cong \frac{\mu_0 I}{2\pi r} +$$

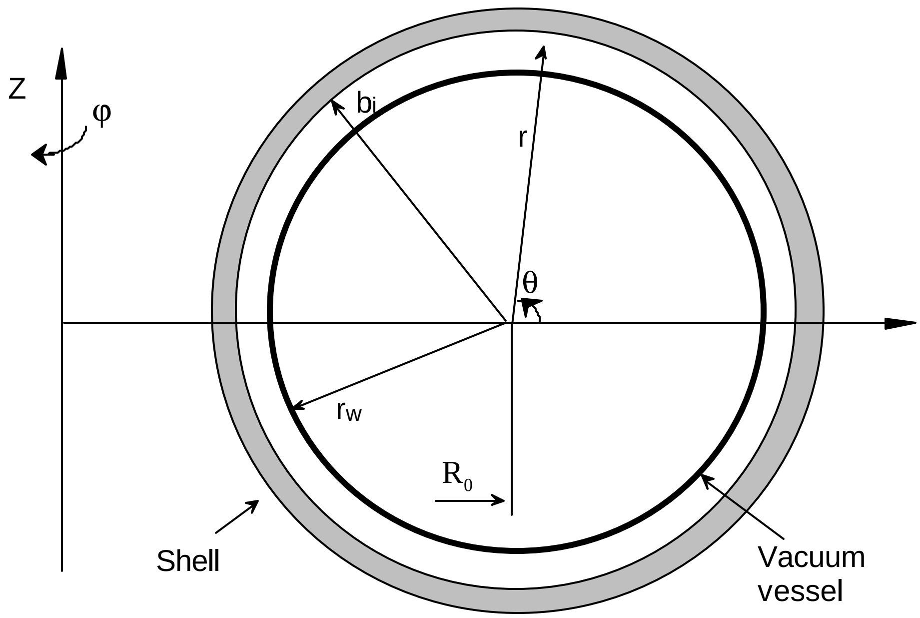
$$+\cos\theta\left[\frac{\mu_0 I}{2\pi r^2}\Delta H(a,\varphi) + \frac{\mu_0 I}{4\pi R_0}\left[\left(\Lambda(a,\varphi)+\frac{1}{2}\right)\left(1+\frac{a^2}{r^2}\right) - 1 + \ln\frac{r}{a}\right] + \frac{B_{\varphi 0}}{R_0}\frac{d\Delta V(a,\varphi)}{d\varphi}\right]$$
$$+\sin\theta\left[\frac{\mu_0 I}{2\pi r^2}\Delta V(a,\varphi) + \frac{\mu_0 I}{4\pi a}\lambda_V(a,\varphi)\left(1+\frac{a^2}{r^2}\right) - \frac{B_{\varphi 0}}{R_0}\frac{d\Delta H(a,\varphi)}{d\varphi}\right]$$

$$B_\varphi(r,\theta,\varphi) = \frac{1}{R}\frac{\partial\chi}{\partial\varphi} \cong \frac{\mu_0 I_T}{2\pi R} +$$

$$-\frac{1}{R_0}\cos\theta\left[\frac{\mu_0 I}{2\pi r}\frac{d\Delta V(a,\varphi)}{d\varphi} - r\frac{B_{\varphi 0}}{R_0}\frac{d^2\Delta H(a,\varphi)}{d\varphi^2} + \frac{\mu_0 I}{4\pi a}\left(r+\frac{a^2}{r}\right)\frac{d}{d\varphi}\lambda_V(a,\varphi)\right] +$$
$$+\frac{1}{R_0}\sin\theta\left[\frac{\mu_0 I}{2\pi r}\frac{d\Delta H(a,\varphi)}{d\varphi} + r\frac{B_{\varphi 0}}{R_0}\frac{d^2\Delta V(a,\varphi)}{d\varphi^2} + \frac{\mu_0 I}{4\pi a}\left(r+\frac{a^2}{r}\right)\frac{d}{d\varphi}\lambda_H(a,\varphi)\right]$$

The radial field contains also another term which quantifies local curvature effects of the vacuum magnetic surfaces

ESTIMATE OF THE PLASMA SHIFT IN RFX



R_0=2m
b_i= 0.538m
r_W =0.457m

<u>Sensors on the innner surface of the shell</u> $r = b_i$:

- 8 poloidal flux loops (axisymmetric equilibrium)
- 2 toroidal arrays of 72 Bφ coils
- 2 poloidal arrays of 16 Br , Bθ coils

4 toroidal flux loops on the external surface of the vacuum vessel $r=r_f$ =0.5m

Instrumented tile on the inner surface of the vessel $r=r_W$ located at $\varphi = 305^\circ$, $\theta = 19^\circ$.

In RFX B_φ is the component measured in the most accurate way (toroidal arrays of probes at r=b) $\rightarrow$ **simplified procedure** to estimate the plasma shift:

$$B_{\varphi,m=1} \approx -\frac{1}{R_0}\cos\theta\left[\frac{\mu_0 I}{2\pi b}\frac{d\Delta V(a,\varphi)}{d\varphi} - b\frac{B_{\varphi 0}}{R_0}\frac{d^2\Delta H(a,\varphi)}{d\varphi^2}\right]+$$

$$+\frac{1}{R_0}\sin\theta\left[\frac{\mu_0 I}{2\pi b}\frac{d\Delta H(a,\varphi)}{d\varphi} + b\frac{B_{\varphi 0}}{R_0}\frac{d^2\Delta V(a,\varphi)}{d\varphi^2}\right].$$

The method is based on the assumption that the terms l are less important than the perturbed shift.

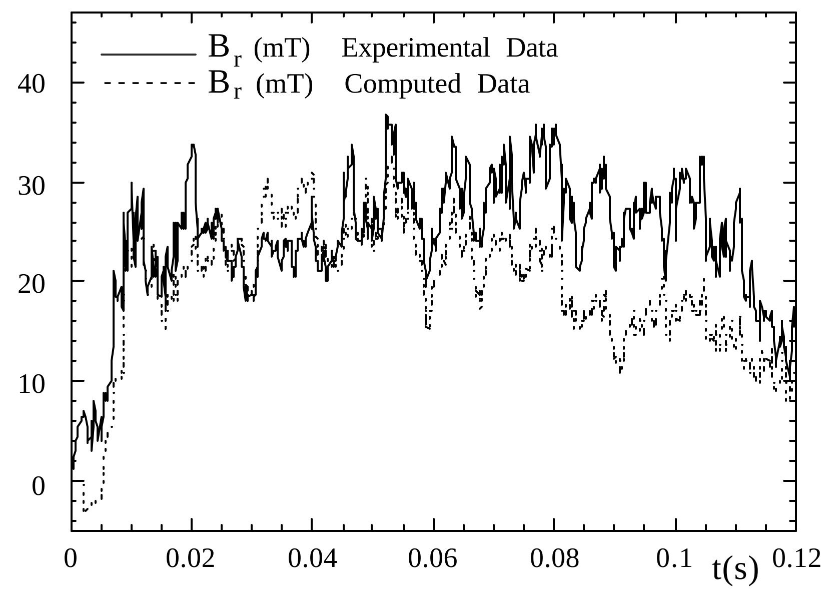
Locked mode appear as a non-axysimmetric equilibrium. Force balance $\rightarrow$ l small

Check: analysis of the radial field at the vacuum vessel

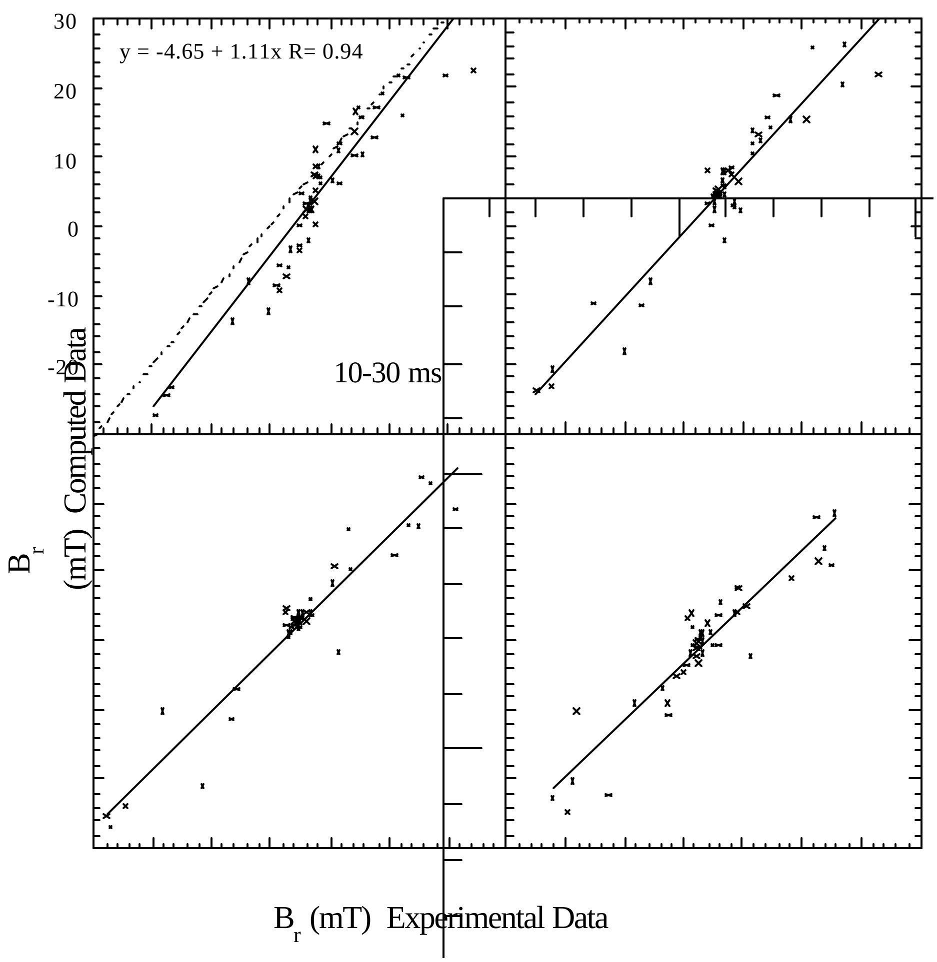
For a limited number of shots the radial field has been measured close to the plasma, at the inner surface of the vacuum vessel (r $=r_w$), by an instrumented tile.

$$B_r(r_W,\theta,\varphi) \cong \cos\theta\left[\frac{\mu_0 I}{2\pi r_W^2}\Delta V(a,\varphi) + \frac{B_{\varphi 0}}{R_0}\frac{d\Delta H(a,\varphi)}{d\varphi}\right]+$$

$$\sin\theta\left[-\frac{\mu_0 I}{2\pi r_W^2}\Delta H(a,\varphi) + \frac{B_{\varphi 0}}{R_0}\frac{d\Delta V(a,\varphi)}{d\varphi}\right].$$



B r (mT) Experimental Data
B r (mT) Computed Data
40
30
20
10
0
0
0.02
0.04
0.06
0.08
0.1
t(s)
0.12



30
20
10
0
-10
y = -4.65 + 1.11x R= 0.94
10-30 ms
B r (mT) Computed Data
B r (mT) Experimental Data

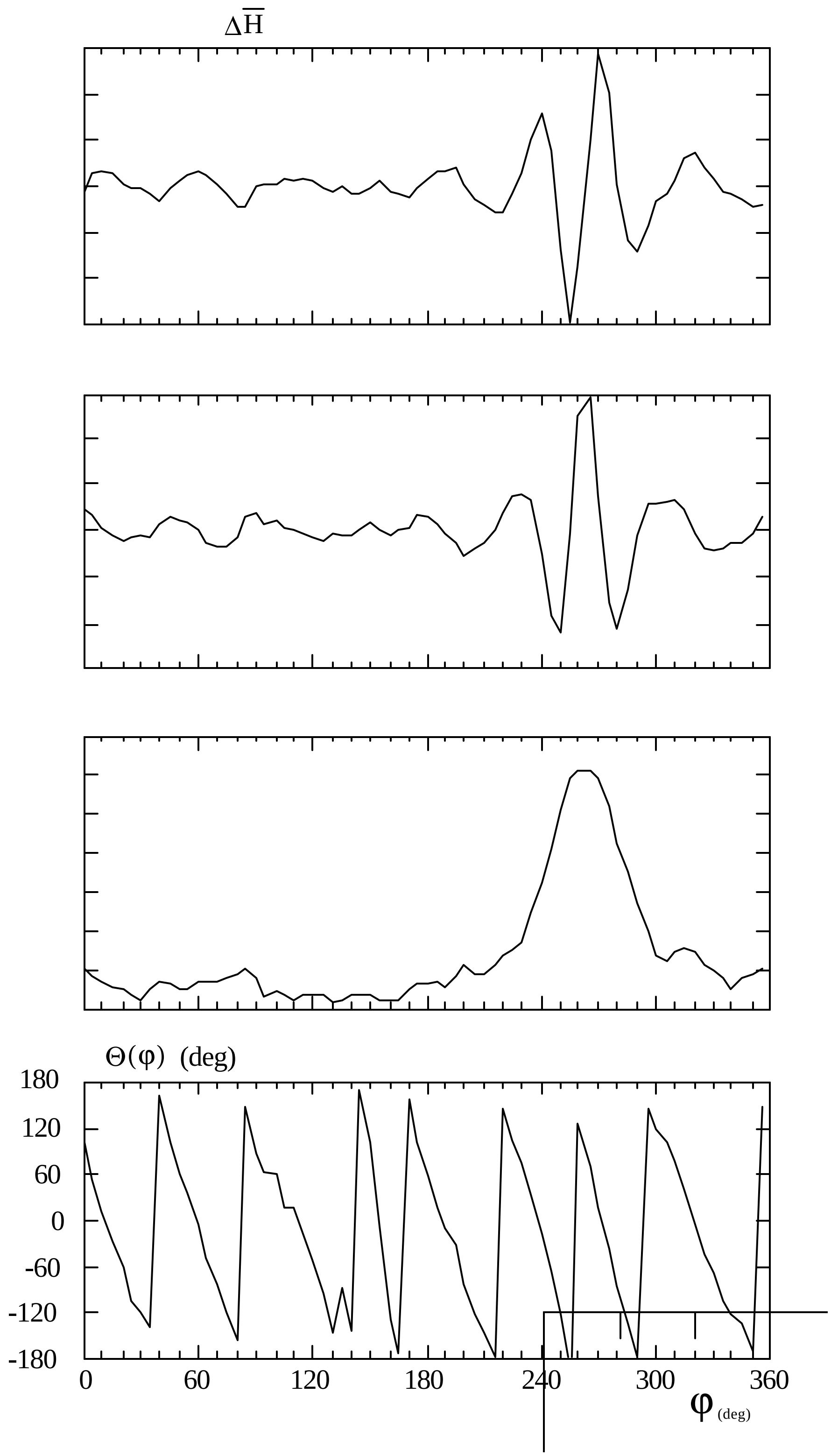
Plasma surface m=1 distortion

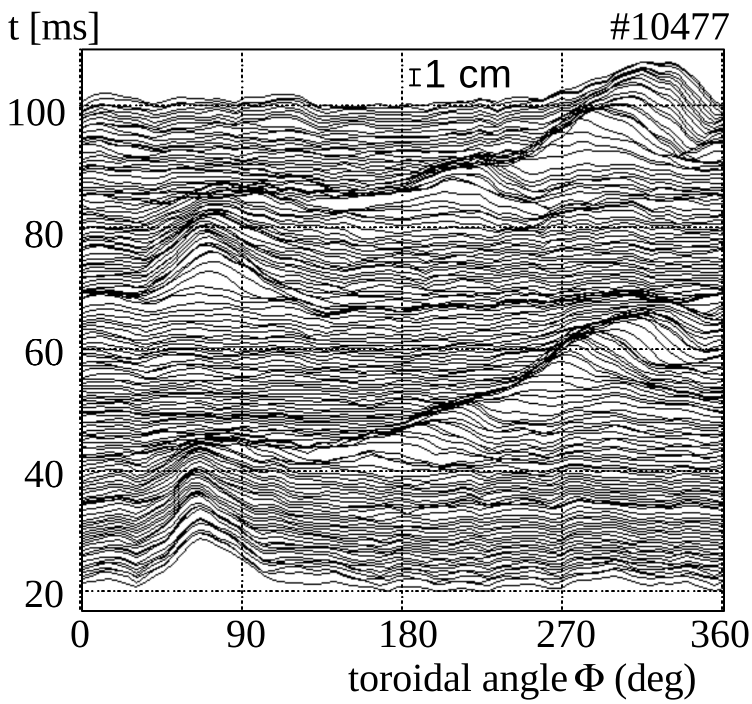
$\Delta H(\varphi); \quad \Delta V(\varphi);$

$$\Delta(\varphi) = \left[(\Delta H(\varphi) - \Delta\overline{H})^2 + \Delta V^2(\varphi)\right]^{1/2}$$

$$\Theta(\varphi) = \tan^{-1}[\Delta V(\varphi)/(\Delta H(\varphi) - \Delta\overline{H})]$$

$$\varphi_{lock}(t) = \max(\Delta(\varphi, t))$$


ΔH̄
Θ(φ) (deg)
180
120
60
0
-60
-120
-180
0
60
120
180
240
300
360
φ (deg)



Superimposition of the waveforms of the function $\Delta(j)$ taken between 10ms and 90ms with a time interval of 1ms, for a discharge characterized by a rotation of the locked mode.

THE M=0 HARMONIC

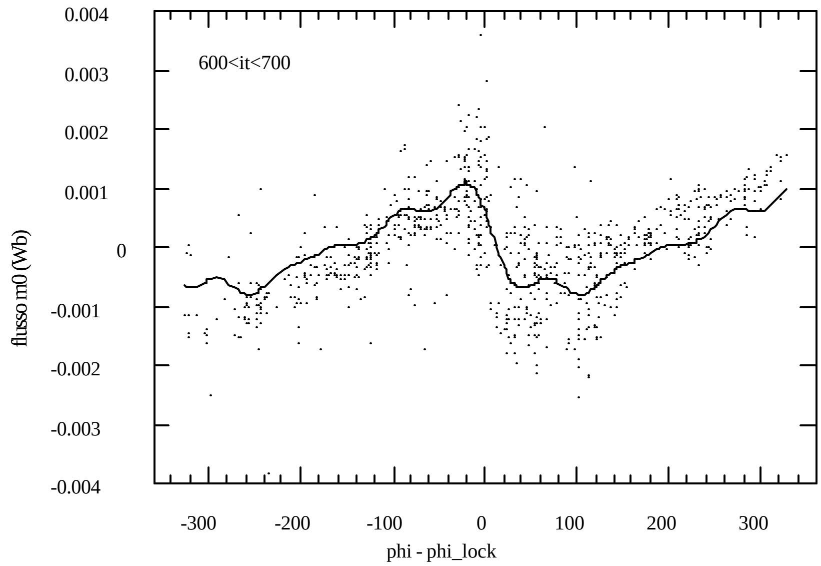
Boundary conditions in RFX

- An estimate of the typical profile of Br (m=0) derived from a statistical analisys made with 4 toroidal flux loops on the external surface of the vacuum vessel $r=r_f=0.5$m
- The typical profile of Bφ (m=0) measured at $r=b_I.=0.538$m

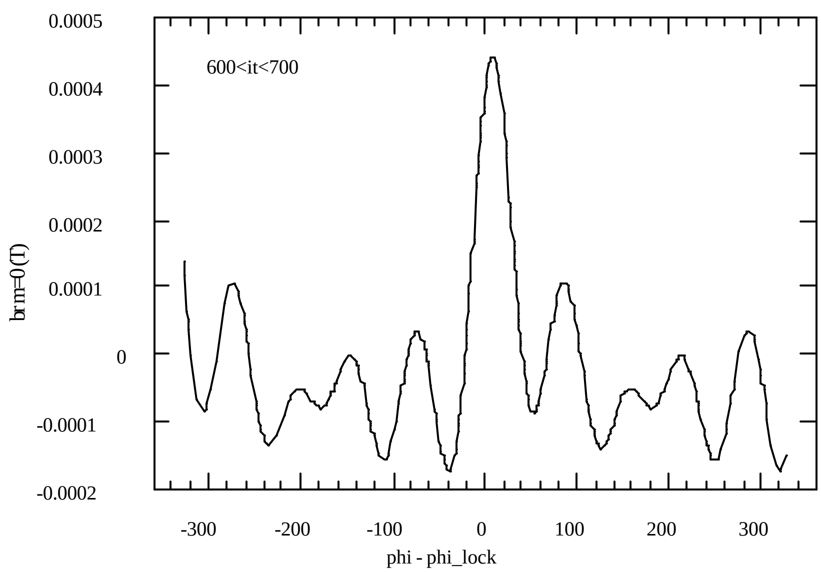
Taking into account the statistical character of the profile we use for the boundary conditions the following calculations are intended to be valid not on the single shot, but as averages over a large number of discharges

Toroidal flux perturbation

Ψ_T (Wb)

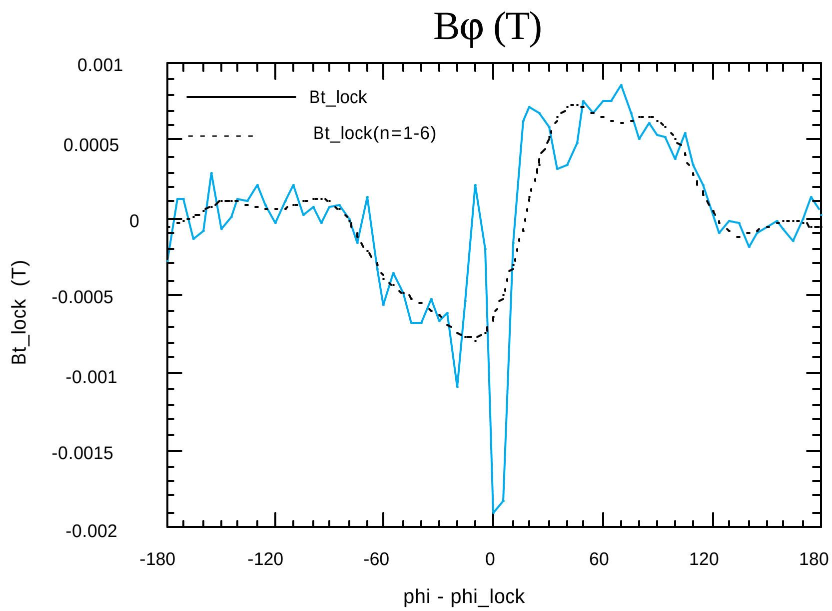


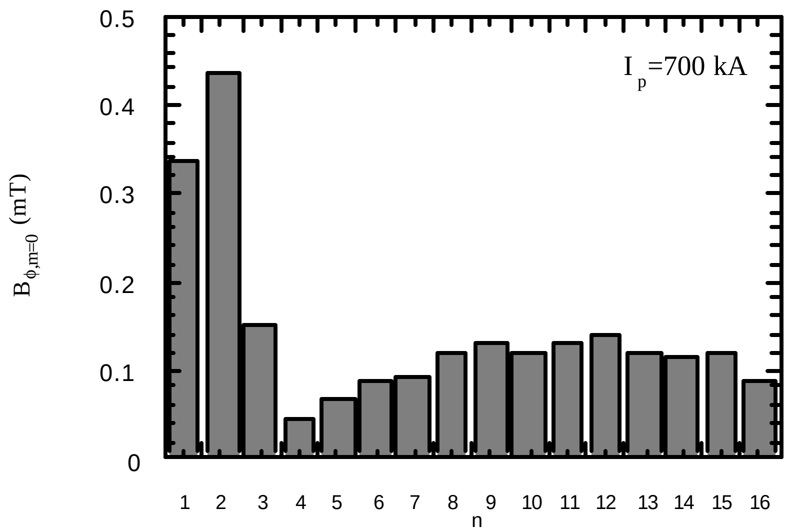
$$B_{r,m=0}(r,\varphi) \cong -\frac{1}{2\pi r R_0}\frac{\partial \Psi_{T,m=0}(r,\varphi)}{\partial \varphi}$$



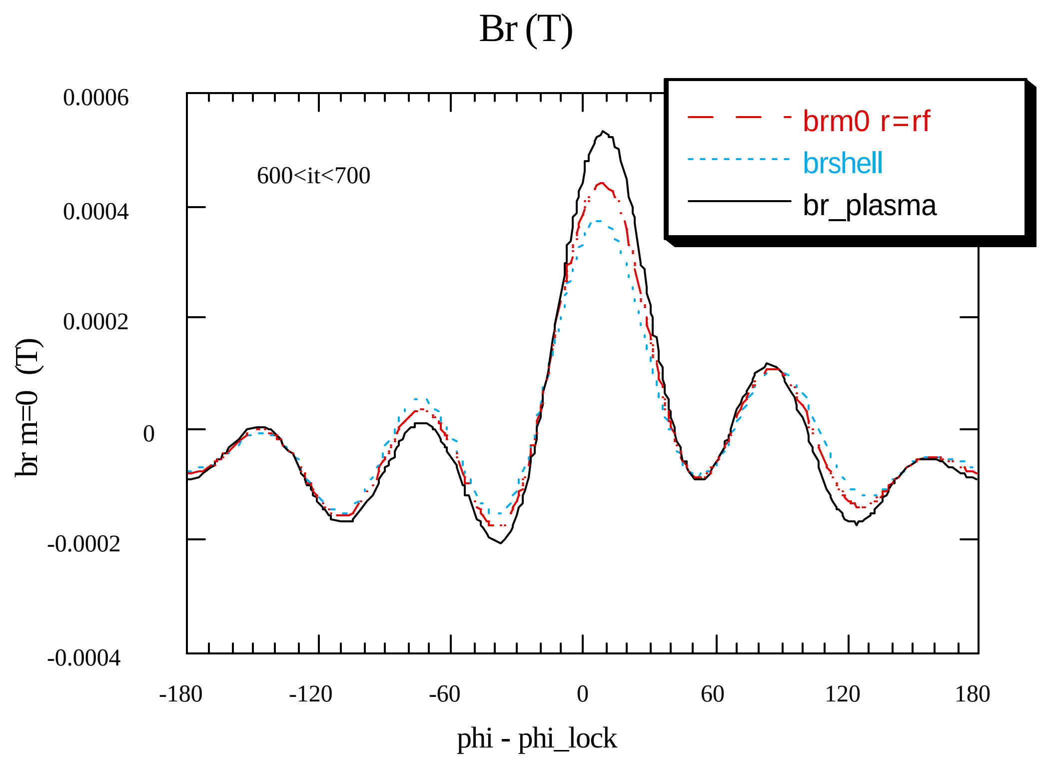
Toroidal field m=0 perturbation

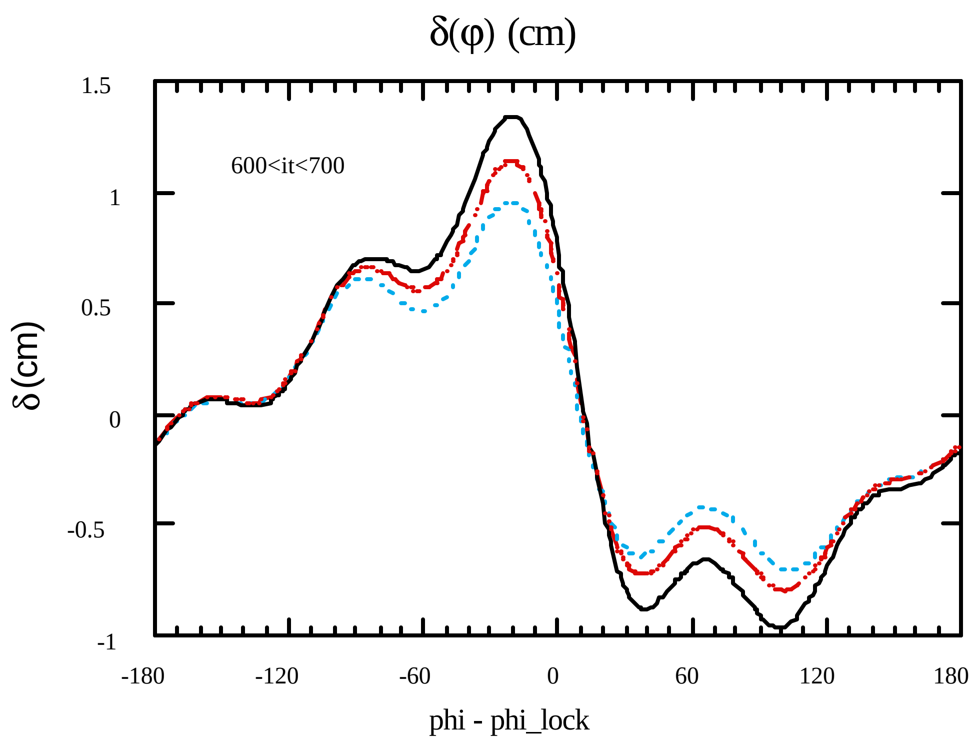
A sistematic contribution due to current that circulates on the shell is eliminated taking the average over a wide number of the signal $B_\varphi(\varphi-\varphi_{lock})$.





The phases of these m=0 modes are consistent with those of the radial field

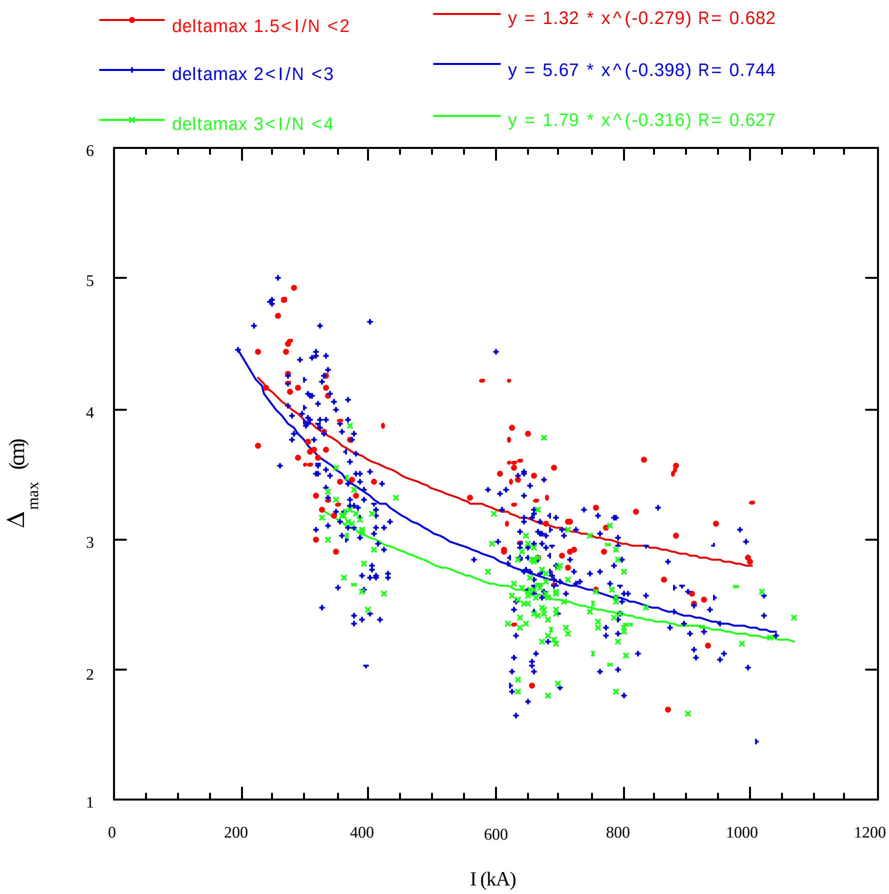




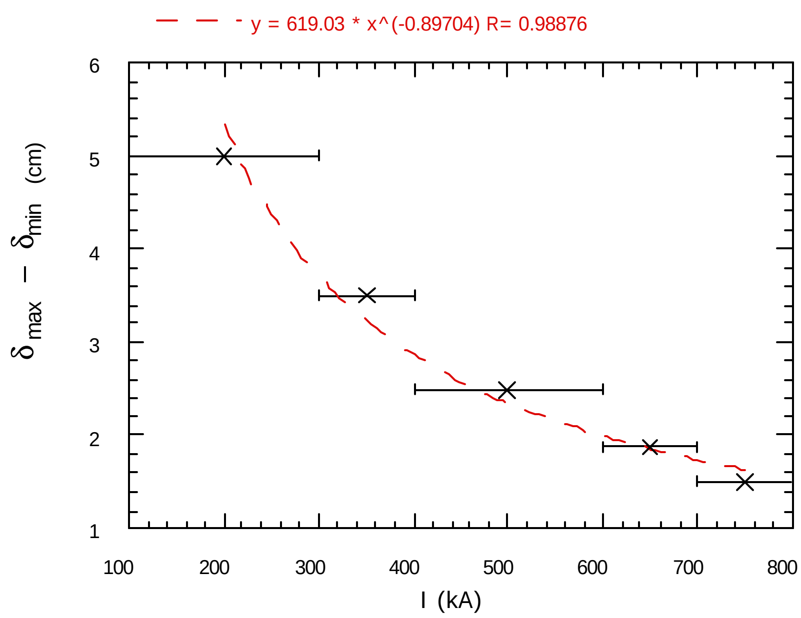
The m=0 bulging of the plasma surface, although not negligible, is **less important** than the m=1 kink

Empirical scaling with I of the plasma perturbation

m=1



m=0



Conclusions

- The reconstruction of the vacuum magnetic surfaces in a toroidal device, based on the measurements of a system of probes located outside the plasma, has been discussed with approximate analytical methods, under the hypothesis that the toroidal and poloidal asymmetries of the configuration can be treated as a perturbation.

- The key information to obtain the non-axisymmetric shape is the radial magnetic field $B_r(r,q,j)$.

- The vacuum radial magnetic field can be computed solving the Laplace equation for the magnetic scalar potential, with suitable boundary conditions.

- Imposing the boundary conditions at the plasma surface generalized Shafranov expression for the vacuum magnetic field are obtained.

- This method has allowed studying the magnetic distortion induced by the RFX 'locked-mode' which consists of a superposition of MHD dynamo modes, mainly $m=0,1$ locked in phase together and to the wall.

- The m=1 perturbation appears as a global helical kink of the plasma column, with a maximum amplitude of about 3÷4cm, weakly decreasing with I

- The m=0 perturbation shows a'jump' in corrispondence of the LM, which strongly scales with I.

[1] Zanca P., Martini S., *"Reconstruction of the magnetic surfaces in a RFP with non-axisymmetric perturbations", Plasma Phys. Contr. Fus. Vol.41 n.10 (1999)*