Lessons Learned from Twenty Years of Chaos and Complexity

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In this article I'll describe the different approaches researchers have taken to understanding the world, make some general observations about the prospects and limitations of their methods, and share some of my views about the future of humanity. It will necessarily be personal and somewhat subjective, and thus probably controversial.

Either explicitly or implicitly, most people, both scientists and non-scientists, are trying to understand the world by making models. Some people have a model in which events are determined by God or perhaps by the position of the planets at the moment of one's birth. A model is a simplified description of a complicated process, ideally amenable to mathematical analysis. However, as the late George Box says, "all models are wrong, but some are useful." Furthermore, the usefulness of a model may not relate to how realistic it is. A simple model is usually more informative and sometimes more predictive than one that includes every effect that one can imagine.

Typically a model involves one or more agents. Although "agent" suggests a person, it could also be a whole society, an industry, an organism, a neuron, or even an individual atom. Agents are exposed to stimuli and exhibit corresponding responses. Sometimes we know the stimuli and are trying to determine the response; other times we observe an action and seek to understand its cause. Science could be defined as the study of such cause-effect relationships.

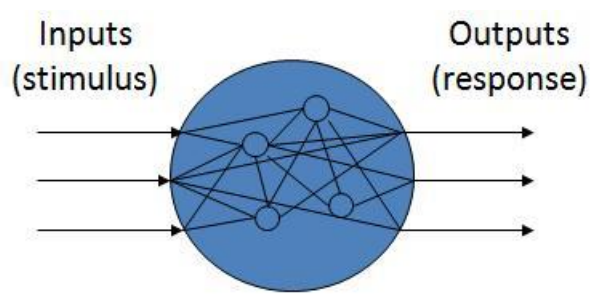

Fig. 1. Science is the study of cause-effect relationships for agents, whose internal workings usually involve other agents.

Consider an example. Somewhere I read that people who floss every day live six years longer than those who don't. The flossing is the stimulus, and the increased longevity is the response. The agent could be an individual, or it could a statistical statement about a whole society.

In fields like physics, we have the luxury of going into the laboratory and doing a controlled experiment on the agent. Even psychologists experiment with human subjects, but more often, when the agent is something like a galaxy, a society, or an economy, the best one can do is to make observations, attempting to correlate stimuli with responses. The difficulties are a paucity of data, a lack of adequate control, and the inability to distinguish correlation from causality. Those who floss are probably also engaging in other healthy activities.

A third approach is to use reductionism, in which one looks at the inner workings of the agent, where other simpler agents are found, and then try to develop a theory relating the response to the stimulus. Scientists are sometimes attacked for their theories by people who equate “theory” with “speculation” and who instead want to know the “facts.” However, theories are much better than facts, since they provide understanding and prediction even outside the realm where they have been tested. If we had a theory for why flossing increases longevity, it might suggest alternate ways to achieve the same or even better result.

I'm glad there are people willing to devote their whole professional career to looking for the Higgs boson or understanding the nervous system of a worm. Reductionism has been a powerful scientific method, but it takes enormous patience, perseverance, and financial and human resources. Furthermore, even a complete understanding of the inner workings of an agent may not shed much light on the emergent behavior of the agent because of the multiple levels of complexity.

A common difficulty is that responses sometimes occur in the absence of any apparent cause, and there are many reasons for such nonstationarity. The agent may be remembering some event in the past, or perhaps the causes are not adequately identified or controlled, or there is noise or measurement error. However, even in a perfect experiment, the agent can exhibit a time-varying behavior due to some internal dynamic even when all the external stimuli are constant -- a common occurrence to which I will return shortly.

The simplest cause-effect relationship is linearity. Linearity does not mean a chain of causality in which *A* causes *B* which causes *C*, and so forth, but rather that the response is proportional to the stimulus. In the flossing example, it means that I would gain about one year of life by flossing weekly, or sixty years by flossing ten times a day. If I accepted the fact about flossing and believed in a linear model, I'd probably be flossing right now.

Furthermore, linearity means that the response to two or more stimuli is the sum of the responses to each individually. Doctor Mehmet Oz, a cardiothoracic surgeon, author, and television personality, claims that those who have 200 orgasms a year live six years longer, which sounds like more fun than all that flossing. Now maybe he means 200 orgasms a year is an optimum, and some of you need to cut back, but my point is that linearity says that I could gain twelve years by appropriately manipulating two parts of my anatomy.

If linear models make such nonsensical predictions, why would one even consider them? First of all, they are simple and provide a good starting point. Secondly, it turns out that most things are linear if the stimulus is sufficiently small. Finally, linear systems of equations can be solved exactly and unambiguously for any number of variables, although, as a practical matter, a computer may be required if the system is large.

It often happens that an agent is stimulated by its own response in a feedback loop, either directly or indirectly through other agents. Thus the effect becomes the cause, and the cause becomes the effect, like the chicken and the egg. The feedback can be either positive (reinforcing the response) or negative (inhibiting it). In such a case, time-varying dynamics can occur because of the inevitable time delay around the loop, and that time delay determines the time scale for the dynamics.

In a linear system with feedback, only four things can happen. Negative feedback leads to exponential decay or a decaying oscillation, while positive feedback leads to exponential growth or a growing oscillation. Positive feedback implies a source of energy or other resource from outside the system. A public address system exhibiting audio feedback will go silent if the power is removed. These four linear behaviors are rarely seen, especially unlimited exponential growth, because resources are limited and nature is not linear.

There are many possible nonlinearities. In two simple examples, the response increases monotonically with the stimulus but either slower than linear (diminishing returns) or faster than linear (economy of scale). An example of a mathematical function that is slower than linear is the square root, and one that is faster than linear is the square. I would argue that the former is more common since the response usually cannot increase without bound. Even if I could gain six years by flossing daily, it's unlikely that I could gain 144 years by flossing hourly or by having 13 orgasms a day. As someone said, "too much of anything is bad; otherwise it wouldn't be too much."

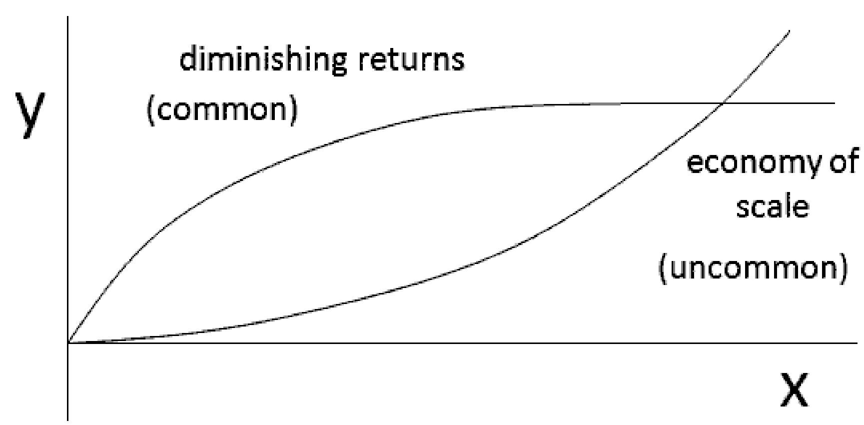

Fig. 2. Two simple examples of nonlinearities, one slower than linear and the other faster than linear.

Nonlinear agents with feedback can exhibit a wide variety of dynamics including the four linear behaviors already mentioned. They can have multiple stable equilibria. They can have stable periodic cycles. They can exhibit quasiperiodicty, which means a combination of periods. They can have bifurcations in which a small change in a parameter causes a completely different dynamic -- what Al Gore and others call a "tipping point." They can exhibit hysteresis, a form of memory in which the original behavior cannot be recovered after a bifurcation without making a large change in the opposite direction. They can have coexisting (or hidden) attractors, meaning that different dynamics are possible even for a given set of conditions, depending on the past history of the system. And, of course, they can exhibit chaos in which a small change in the initial condition completely changes the future.

Most systems in the real world involve large networks of nonlinearly interacting agents. The ecological system, the climate system, the political system, and the economic system each involve numerous agents and are strongly coupled to one another. Of necessity, most scientists are studying a small part of a much larger network, hoping that the part not being studied can be treated as a fixed external stimulus. I think this often leads to erroneous conclusions and predictions, as does the implicit assumption of linearity and the disregard of feedback loops.

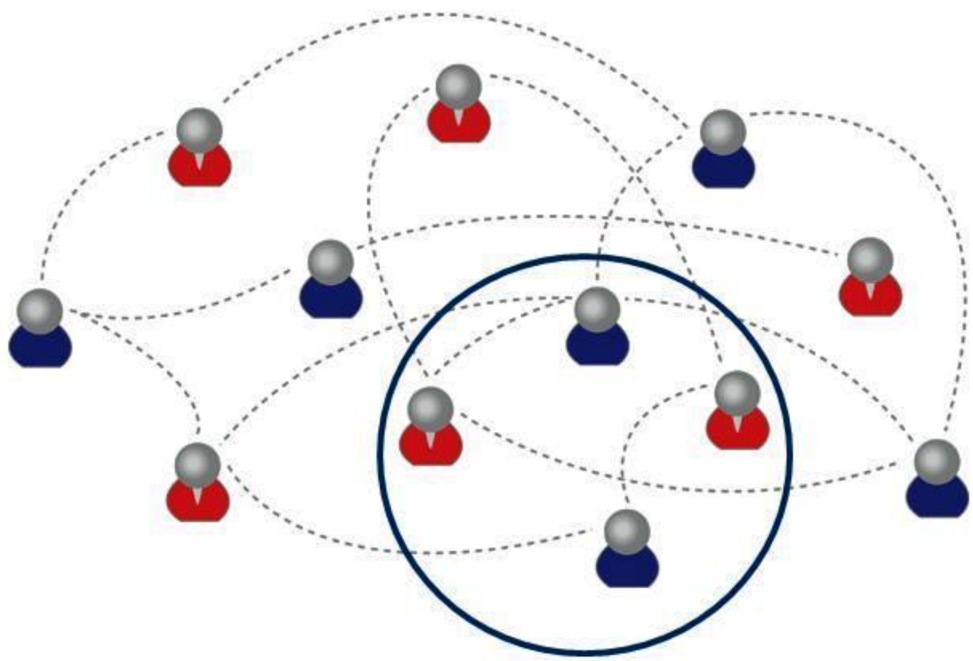

Fig. 3. Most scientists, of necessity, are studying a small part of a much larger network, hoping that the part not being studied can be treated as a fixed external stimulus, often leading to erroneous conclusions and predictions.

For example, if some species of animal consumes some species of plant as its primary food supply, and the abundance of that plant is suddenly reduced to half, we might naively assume that half the animals would die. However, it is much more likely that they would find a different source of food somewhere. Similarly, if global warming causes the sea level to rise a meter over the next century, it's unlikely that the hundred million people who now live along the coast will drown as a result, and much more likely that they (or rather their descendants) will simply migrate to higher ground, or perhaps they will build some simple dikes as the Dutch have done.

An alternate approach is to characterize the general behaviors of large nonlinear networks without regard to what they are modeling. This is an extension of the method used by mathematicians to characterize the nonlinear dynamics of simple systems. The task is made difficult (and interesting) by the fact that the architecture of a network (the connection strengths between the agents) can change in time even while the network is exhibiting dynamics, and the two types of dynamics are coupled. This distinction is sometimes called the dynamics OF the network as opposed to the dynamics ON the network. The neurons in the brain slowly reconnect even while the brain is actively performing tasks and in response to those activities. Curiously, an evolving network can always be exactly represented by a (sometimes much) larger network with static connections. What we need is a set of laws governing the behavior of large networks analogous to the laws of thermodynamics that describe the behavior of gases without the necessity of knowing what the individual molecules are doing or why or even that the gas is made up of molecules.

If I may digress for a moment, I would like to mention one accomplishment of which I'm especially proud. Twenty years ago, I became interested in the question of what is the simplest network that is capable of exhibiting chaos. One would think that question had long ago been asked and answered, but apparently not. I didn't originally think of the question in that way, but rather I was trying to find the simplest ordinary differential equation whose solution is chaotic, and it was only in preparing this lecture that I realized it was the same question. It has long been known that at least three agents are required and that at least one of them must be nonlinear, but I was able to show that only three feedback loops are required and how they are arranged (Sprott, 1997). Two years later Stefan Linz and I found another equally simple arrangement (Linz & Sprott, 1999).

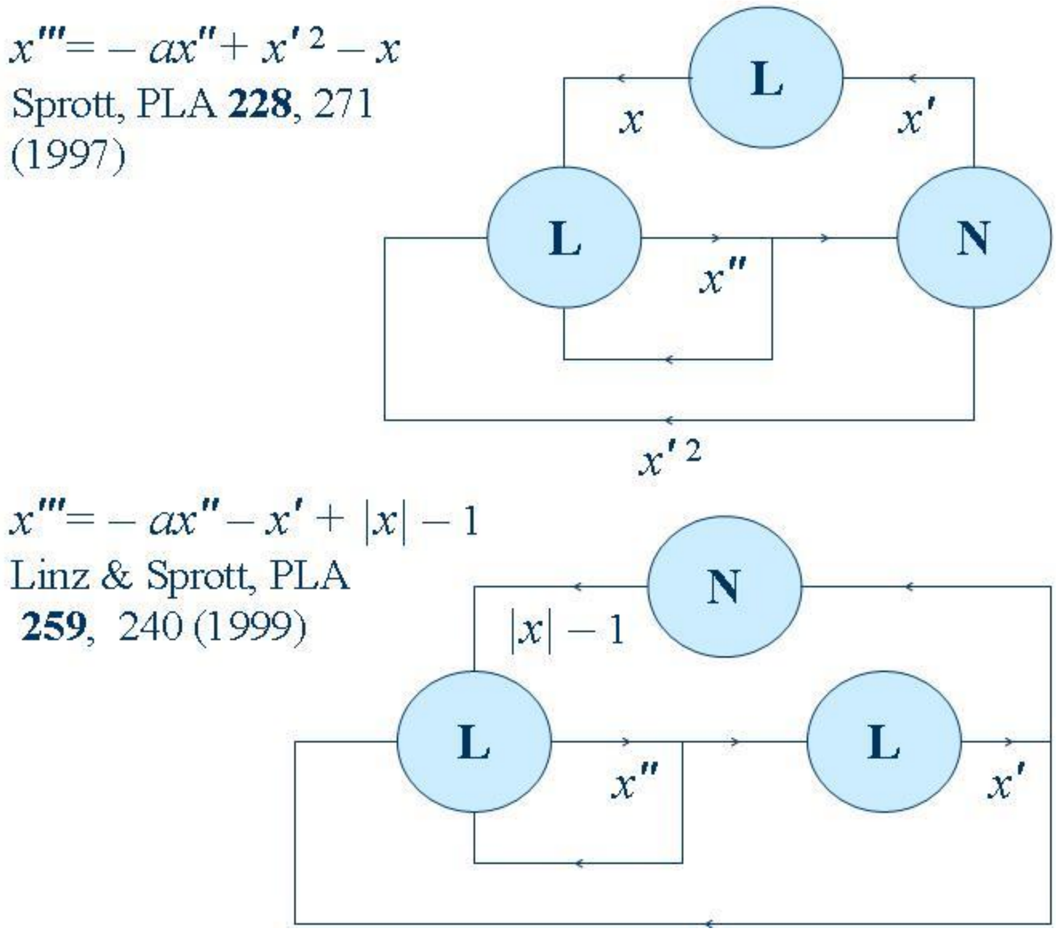

Fig. 4. The simplest nonlinear networks that are capable of exhibiting chaos.

Large nonlinear networks are appropriate models of complex adaptive systems of the type that occur throughout nature, and much has been learned recently about their behavior. In particular, they are usually chaotic, although only weakly so, and thus they are inherently unpredictable but sensitive to small changes in both the state of the system and the parameters, and thus potentially easily controllable. More interestingly, such systems can self-organize, adapt, and learn -- qualities we normally associate with human intelligence, but that are observed in physical systems as well. Witness the organization of the Universe into galaxies and stars and planets that ultimate gave rise to life on Earth.

In recent years, many people have made dire predictions, especially regarding the climate, the economy, and the ecology, but I am more optimistic than most about our future for five fundamental reasons:

1) Negative feedback is at least as common as positive feedback, and it tends to regulate many processes.

2) Most nonlinearities are beneficial, putting inherent limits on the growth of deleterious effects.

3) Complex dynamical systems self-organize to optimize their fitness.

4) Chaotic systems are sensitive to small changes, making prediction difficult, but facilitating control.

5) Our knowledge and technology will continue to advance, meaning that new solutions to problems will be developed as they are needed or, more likely, soon thereafter in response to the need.

Whether it's fusion reactors, geoengineering, vastly improved batteries, self-driving cars, halting of the aging process, memory implants, de-extinction, or some other game changer, things may get worse before they get better, but humans are enormously ingenious and adaptable and will rise to the challenge of averting disaster.

This is not a prediction that our problems will vanish or an argument for ignoring them. On the contrary, our choices and actions are the means by which society will reorganize to become even better in the decades to follow, albeit surely not a Utopia.

References

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