A chaotic model of sustaining attention problem in attention deficit disorder

G. Baghdadi a, S. Jafari a,⇑, J.C. Sprott b, F. Towhidkhah a, M.R. Hashemi Golpayegani a

a Biomedical Engineering Faculty, Amirkabir University of Technology, Tehran, Iran
b Department of Physics, University of Wisconsin, Madison, WI 53706, USA

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Abstract
The problem of keeping an attention level is one of the common symptoms of attention deficit disorder. Dopamine deficiency is introduced as one of the causes of this disorder. Based on some physiological facts about the attention control mechanism and chaos intermittency, a behavioral model is presented in this paper. This model represents the problem of undesired alternation of attention level, and can also suggest different valuable predictions about a possible cause of attention deficit disorder. The proposed model reveals that there is a possible interaction between different neurotransmitters which help the individual to adaptively inhibit the attention switching over time. The result of this study can be used to examine and develop a new practical and more appropriate treatment for the problem of sustaining attention.

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1. Introduction

Attention deficit disorder (ADD) is one of the most common neurobehavioral disorders. It is usually first diagnosed in childhood, and its symptoms often last into adulthood. A person with ADD often avoids, dislikes, or does not want to do things that take a lot of mental effort for a long period of time. He/she is often easily distracted, usually has trouble keeping attention on tasks or play activities and frequently switches from one activity (mental or physical) to another [1], whereas a person without such a disorder can keep his/her attention for more time in the same situation [2]. Therefore, switching between different activities and inability to maintain attention are not because of sudden changes or noise in the environment. It appears that there is an inherent switching in the attention controlling mechanism of people with attention deficit disorder. This kind of inherent switching is a universal feature of many natural systems, especially neurons [3].

The attention switching symptom of ADD is behaviorally similar to intermittent chaos. Intermittency is a property of a chaotic system in which the dynamics switch back and forth between two qualitatively different behaviors (e.g. periodic to chaotic) even though all control parameters remain constant and no significant external noise is present. Wandering between these behaviors will continue while the system is in the intermittency mode [4]. In other words, in chaotic intermittency, a system cannot preserve its ordered manner and switches repeatedly to chaos. Similarly, people with inattentive type of ADD have difficulty keeping their attention on a task, and they switch between different activities.

The study of chaotic dynamics has received increasing attention and has provided a promising method for studying biological systems and signals [5–9]. The studies conducted so far have shown that chaos plays an essential role in neural
systems analysis, and brain signals have deterministic chaotic properties [7]. Biological system modeling is another field of study that exploits chaos theory [10–13].

Based on the properties of chaos intermittency and the physiological facts about the human attention system, a novel top–down behavioral model of the attention controlling mechanism in people with ADD is proposed in the current study. Top–down dynamical models start with an analysis of those important aspects of behavior that are robust and reproducible. The top–down approach is a more speculative, big-picture view. The model should predict how the behavior evolves with different changes in the environment [14].

In previous studies, different animal and computational models were presented which show some other problems of ADDs [15–17]. For instance, Balkenius and Björne [15] presented a robot model of attention deficit hyperactivity disorder (ADHD) which can reproduce some of the behaviors of people with ADHD. They show the slow reaction time for people with ADHD and predict the possible working memory impairment in these people. However, no simulation of the sustaining attention problem was done in that study [15]. Brown (2001) developed a model of ADD based on the physiological facts and his experiments with children who have ADD/ADHD. Brown’s model describes the complex cognitive functions impaired in ADD Syndrome [16]. There are also some animal models of ADHD in which their goal is to show parts of the brain that are involved in ADHD [17].

In the current study, a new model is proposed whose main goal is the investigation of the frequent attention switching problem in ADDs. Using chaos theory to model an abnormality in people with ADD is one of the novelties of the current study. This model has the ability to show the effect of interaction between attention system components in keeping an individual’s attention on a task. It can also predict some possible causes of the attention switching problem.

Section 2 outlines the information about the main parts of the brain that are associated with attention deficit disorders. Section 3 contains details about the proposed model’s components. The results and discussion of this study are presented in Section 4 followed by the conclusion in Section 5.

2. The physiological background

The frontal lobe of the brain is a very important part of a complex cognitive processing system. It has many connections to different areas of the brain. Research on the brain function of people with ADD has shown that frontal lobe dysfunction may cause the appearance of ADD symptoms [18]. The frontal cortex has an important role in controlling attention level, focusing, restraint, and patience. When this part of the brain does not work well, signs of distraction, lack of restraint, impatience, and lack of attention to detail are seen in the person [19]. The frontal cortex also plays an important role in the excitation/inhibition balance in information processing [20]. In people with ADD, frontal lobe dysfunction reduces the inhibitory power of the brain, and they have difficulties in inhibiting their attention switching [21–25]. However, there are some recent studies that claimed that the excitatory brain components can also affect ADD [26–29].

It has been reported that a neurotransmitter called dopamine has a considerable effect on frontal lobe function [30]. Several important diseases of the nervous system are associated with dysfunctions of the dopamine system [31,32]. ADD is also believed to be associated with decreased dopamine activity [33]. Dopamine plays a major role in the brain system that is responsible for reward-driven learning [34]. Every type of reward that has been studied and also stressful situations increase the level of dopamine transmission in the brain [35,36].

3. The proposed model

According to the previous discussion, a nonlinear neural network is proposed (Fig. 1) to model the ADD attention switching behavior. Choosing appropriate values for the parameters leads to chaotic behavior.

The model input is composed of two main parts: First, sensory information is received from the subject, on which the person is asked to concentrate. Second, feedback information is extracted from analysis and perception of sensory information. The sensory cortex receives the input information and sends it to the inhibitory and excitatory parts of the brain with an amplification factor of \( w_i \). These parts are attributed to the frontal cortex.

The output of this network can be computed as follows:

\[
\text{Processing an perception (frontal cortex)}
\]

\[
\text{Input} \quad \begin{align*}
\text{Sensory cortex} & \quad \text{Output} \\
\text{Input} & \quad \text{Output}
\end{align*}
\]

\[
\begin{align*}
\text{Sensory cortex} & \quad \text{Input} \\
\text{Processing an perception (frontal cortex)} & \quad \text{Output}
\end{align*}
\]

\[
\text{Fig. 1. A behavioral chaotic neural network model of ADD. } E(x) \text{ and } I(x) \text{ are the activation functions of two neurons whose outputs are respectively multiplied by } (B) \text{ and } (A). \text{ Both } E(x) \text{ and } I(x) \text{ are hyperbolic tangent functions. } E(x) \times (\text{–A}) \text{ models the inhibitory brain action and } E(x) \times (+B) \text{ models the excitatory brain action. (A), (B), and } w_i \text{ are values that amplify the output of the neurons.}
\]

In this model, $E(x)$ and $I(x)$ are the activation functions of two neurons whose outputs are respectively multiplied by ($B$) and ($A$). Hyperbolic tangent activation functions are considered for both $I(x)$ and $E(x)$. However, the output of $I(x)$ enters the output neuron with a negative value that models the inhibitory brain action, and the output of $E(x)$ enters the output neuron with a positive value that models the excitatory brain action. All coefficients ($A$, $B$, $w_1$, and $w_2$) are associated with the brain synapses’ weights that are regulated by the release of different neurotransmitters. The value of these coefficients can be varied to produce different behaviors that will be discussed in the next sections. Since the value of ($A$) shows the power of inhibitory brain action, inappropriate values of ($A$) that can be correlated with the amount of dopamine required to cause inhibitory problems in people with ADD.

The result of executing all excitatory or inhibitory processes will be the individual’s perception from the environment that will produce a level of attention and will affect the input information. Neural studies have shown that attention increases the firing rate of neurons and also makes more neurons fire simultaneously \cite{37,38}. Consequently, it follows that more attention leads to more neuron output energy over a period of time.

In the present study, the root mean squared (RMS) value of the output (which can be considered proportional to its energy) is considered as the “attention level” over a period of time. Therefore, the “attention level” in a specified time window is calculated as follows:

$$\text{attention level} = \sqrt{\frac{1}{(n_2 - n_1 + 1)\sum_{n=n_1}^{n_2} \text{out}(n)^2}} \quad (2)$$

where $n_1$ and $n_2$ are respectively the first and last time samples of the considered time window and $\text{out}(n)$ is the amplitude of the output in the nth time sample. Therefore, Eq. (2) can be used to calculate the changes in attention level over different time windows.

4. Results and discussion

The proposed model is simple with a few parameters whose values are important for changing the behavior of the system. As was mentioned in the physiological background, many different studies have claimed that dysfunction of the inhibitory part of the brain is responsible for ADD symptoms, and some of them showed that the excitatory brain components are also important in ADD.

Therefore, the analysis of the model is done first considering a fixed value of the coefficients: $B$, $w_1$ and $w_2$; because they do not relate to the inhibitory part. The values of these parameters are chosen by trial and error so that there is an intermittency mode in the model's bifurcation diagram based on different values of the parameter ($A$) for $B = 5.821$; $w_1 = 1.487$; $w_2 = 0.2223$. It should be noted that there are many choices for these values that cause intermittency. The goal of this study is not to investigate the quantitative value of these parameters but rather to show that a small change in the model parameters (which are attributed to the neurotransmitters) can lead to undesired behavior of the system. Actually, all of these coefficients are internal factors (the effect of neurotransmitters) which are automatically set in the brain, and in some disorders these coefficients may change from their optimal values.

Considering the above coefficient values, the bifurcation diagram of this neural network for different values of ($A$) is plotted in Fig. 2.

According to the previous discussion, attention switching, which is one of the common symptoms of ADD, is analogous to intermittency in chaotic systems. Therefore the parameter ($A$) is set at a value for which the model exhibits intermittency. In this mode, the output of the model switches between different behaviors without any changes in the environment (the model parameters). But it is expected that a healthy person can control and sustain his/her attention when there is no disturbance in the environment. In other words, the attention level frequently switches in people with ADD for no apparent reason, whereas a healthy person can keep his/her attention level nearly constant over some period of time. The output of the model at $A = 12.473$ is shown in Fig. 3. At this value, the model is in a periodic window.

According to Fig. 3, the behavior of the system is ordered (periodic) for the selected values; and the attention level (which is calculated by using Eq. (2)) does not change in time. In other words, the attention level is not reduced over time, and the model keeps the level of the attention constant. However, by reducing ($A$) from 12.473 to 12.472, the output changes as shown in Fig. 4 (reducing the value of ($A$) is equivalent to a reduction in dopamine secretion).

According to Fig. 4, it seems that reducing the value of ($A$) causes the system to enter into intermittency, and the attention level alternates over time. Further reduction of ($A$) to $A = 12.461$ changes the output as shown in Fig. 5.

Fig. 5 shows that a small reduction of ($A$) leads to more switching in the output behavior (intermittency) and consequently in the attention level; that is, the model for these parameter values was not able to keep the attention level constant.
over time. This behavioral switching during different time windows occurs without any changes in the parameter values (environmental conditions). This intermittent behavior is analogous to the AD/HD’s attention switching symptom.

Controlling the attention switching in ADHDs is usually done by drug consumption. These drugs increase the level of dopamine in the brain [1]. According to this model, it seems that these drugs push the system from intermittency to periodicity.

4.1. The possible interaction between neurotransmitters

Taking a closer look at the obtained result shows that the dopamine secretion \( A \) does not affect the attention level control alone. It seems that proportionality between dopamine \( A \) and other neurotransmitters \( w_1, w_2 \) and \( B \) is important in
controlling the attention level. The reason behind this claim is that if it is assumed that there is no proportional interaction or coupling between dopamine and other neurotransmitters, then small changes in the amount of dopamine causes a serious attention abnormality, while we know the brain is more robust against small changes in dopamine. Therefore, it is predicted that when the amount of dopamine ($A$) is close to a value that brings the system out from periodic windows, the values of the other parameters (neurotransmitters) will be changed so as to inhibit the system from becoming intermittent. This prediction agrees with recent evidence in the literature which indicates that more neurotransmitters contribute in controlling attention level \[39,40\]. Fig. 6 shows that a small increment in the ($B$) value from 5.821 to 5.826 can compensate for the reduction in the ($A$) value, and consequently, can bring the system out of intermittency.
According to Fig. 6, it seems that the amount of the other neurotransmitters is also important to prevent the attention level alternation over a period of time. Fig. 7 shows the bifurcation diagram of the proposed model for different values of \((B)\) at a fixed value of \((A=12.473)\).

Fig. 7 shows that changes in the value of \((B)\) can also affect the system behavior. However, if it was considered that one parameter in the model is varied and the other is fixed, then the model is very sensitive to the parameter value, and small changes bring the model into intermittency. Therefore, it is likely that there is a mechanism that keeps the system (the mind of the person with ADD) in the intermittent state. A possible mechanism is the interaction and coupling between neurotransmitters. That is, changing the value of one neurotransmitter (e.g. the parameter \((A)\)) will lead to changes in the other

![Figure 6](image1)
![Figure 7](image2)

**Fig. 6.** Top: the model output for \(A = 12.461, B = 5.821; w_1 = 1.487; w_2 = 0.2223;\) Bottom: the model output for \(A = 12.461 B = 5.826; w_1 = 1.487; w_2 = 0.2223\) (bringing the system out of intermittency to periodicity by increasing the \((B)\) value by about 0.005).

**Fig. 7.** The model bifurcation diagram based on different values of the parameter \((B)\), \(A = 12.473; w_1 = 1.487; w_2 = 0.2223\).
neurotransmitter values (e.g., the parameter \((B)\)). Fig. 8 shows different behaviors of the proposed model for simultaneous changes of both parameters \((A)\) and \((B)\).

Fig. 8 demonstrates that the simultaneous influence of two factors \((A)\) and \((B)\) creates a region of parameter space where the system can remain long enough for intermittency to occur. That is, the interaction and coupling between these two neurotransmitters causes the model to be more robust to changes of neurotransmitter values.

The simultaneous effect of two parameters was investigated in this section. However, it is obvious that more parameters are involved in the attentional control system that may change the region of parameters where the system shows different behaviors.

4.2. A possible attenuator in the feedback path

An input/output feedback was considered in the proposed model (Fig. 1). According to this model, it can be predicted that a defect of this path may cause the attention switching problem. In line with this assumption, in this section an attenuator is considered in the feedback path (Fig. 9).

Fig. 10 shows the bifurcation diagram of the proposed model for different values of \((K)\) at a fixed value of the other coefficients.

In a healthy subject, it is expected that the output information feeds back to the input without any flaws \((K = 1)\). According to Fig. 10, it is also observed that the widest periodic window (robust attention level) is around \(K = 1\). If the output information is malformed along the feedback path \((K < 1)\), the subject senses incorrect results from the forward processing of the information. The imperfect perception from the forward stimuli processing cannot motivate the subject to keep his/her attention on the input stimuli and consequently causes attention switching. In this situation, neurofeedback training can help the subject to correct the problem [41]. Fig. 10 shows that for \(K\) values lower than about 0.7, the model shows periodic behavior resulting in a fixed attention level. The condition \(K > 1\) means that adding information to the output signal is not considered in this model, because we could not identify any biological evidence to support this condition, especially in people with attention disorder. Thus the model’s equation is expressed as follows:

\[
out(n + 1) = K \cdot [B \cdot \tanh(w_1 \cdot out(n)) - A \cdot \tanh(w_2 \cdot out(n))]; \quad K \leq 1
\]  

(3)

In this equation, \((K)\) is the attenuation coefficient. This coefficient \((K)\) models the condition that the person could not analyze the input information perfectly and leads to an imperfect perception and consequently a lower attention level. Fig. 11 shows the behavior of the model for two values of \((K)\).
Fig. 9. Considering an attenuator in the feedback path. $K = 1$ is a model of a healthy subject (the output of processing has no changes), $K < 1$ is a model of people with attention deficit disorder (the output information is malformed along the feedback pathway). $K > 1$ models adding extra information to the result of forward processing for which there is not any supporting biological evidence for this condition.

Fig. 10. The model bifurcation diagram based on different values of the parameter $(k)$, $A = 12.473; B = 5.821; w_1 = 1.487; w_2 = 0.2223$.

Fig. 11. The behavior of the model for two values of $(K)$ (feedback gain).
Fig. 11 shows that weakening the feedback gain (reducing the $K$ value from 1 to 0.829) may cause the model to show intermittency (the value of $K = 0.829$ is randomly selected from one of the $K$ values that belong to the chaotic area in the bifurcation diagram (Fig. 10). Therefore, it can result that a defect in the feedback path (which is modeled by an inappropriate value of $K$) may also cause the problem of maintaining attention over a period of time. In other words, it is possible that the value of the neurotransmitters is appropriate, but because of a problem in the frontal cortex, the analysis of the input information is not done perfectly and causes an incomplete understanding of the information (weakening the feedback information) and consequently leads to attention switching.

In the same way as in the previous section, the effect of the $K$ value should be investigated considering the values of the other neurotransmitters (e.g. $A$ and $B$) simultaneously. Fig. 12 reveals how the simultaneous variation of $A$ and $K$ for different values of $B$ changes the model behavior. In this figure, the values of $B$ are randomly selected, and for other values of $B$ the behavior of the model varies similarly.

Fig. 12 shows that changing the value of $B$ can increase or decrease the region of parameter space ($K$ and $A$) over which the system exhibits a certain behavior. For example, for $B = 1.5$, the region where the system is in intermittency is narrower than for $B = 7.5$.

In Fig. 12, the plot for $B = 7.5$ shows that the size of the intermittency region (yellow) is large, and thus keeping the system in the intermittent state is not so difficult. Increasing the value of $B$ (greater than 20) or decreasing its value (less than 7.5) leads to a reduction in the intermittency region. Therefore, it seems that an appropriate range of $B$ is between 7.5 and 20 for modeling intermittency.

Fig. 13 shows how a simultaneous variation of $B$ and $K$ for different values of $A$ changes the model behavior. The model behavior is plotted for six randomly selected values of ($A = 5.5, 8, 13, 18, 23, and 25$), and for other values of $A$, the behavior of the model varies similarly.

According to Fig. 13, the values $A < 13$ or $A > 25$ are not appropriate for modeling attention switching (intermittency) because at these values the intermittency region is so narrow that keeping the model in this state is too difficult. Therefore, there is an upper and lower bound for the range of $A$ to model intermittency. In the other words, the proposed model shows the switching symptom of people with attention deficit disorder in a bounded values of the coefficients.

Fig. 12. The model behavior with a variation of ($A$) and ($K$) for different randomly selected values of ($B$).
4.3. Dopamine secretion rate as a function of time

The studies conducted so far reported that attention switching also occurs in healthy subjects over time (about 40 min) due to fatigue [42]. Therefore, it is tempting to claim that the dopamine secretion rate decreases as a function of time. Thus the value of dopamine (here \( A \)) decreases gradually and after about 45 min (in healthy subjects) reaches a value that leads to attention switching, and other neurotransmitters could not compensate for its reduction. This claim, considering neurotransmitter secretion as a function of time, can also be used for other model parameters. An equation that models the decrease in dopamine secretion rate is

\[
A = a_1 \exp\left(-a_2 \times t\right)
\]  

(4)

Considering such a function for the dopamine secretion, it is claimed that the value of \( a_2 \) in people with ADD is greater than in healthy individuals because the attention span for people with ADD is less than for healthy subjects [43]. However, because of the possible involvement of the other parameters, it seems that the neurotransmitter secretion function is not a fixed function of time, and its dynamic will change in accordance with other events that occur in the system.

Therefore, another prediction about the causes of the attention switching problem is the inappropriate parameter values of the function of the neurotransmitter secretion rate over time.

5. Conclusion

In this research, a behavioral chaotic model of a human's attention controlling mechanism is presented. This model has the capacity to show the sustaining attention problem in people with ADD.

Using the capabilities of the proposed model, it is revealed that the interaction between different neurotransmitters that are involved in sustaining attention can create a self-organized attention controlling mechanism, and it can also reduce the sensitivity of the attentional control mechanism to very small neuro-chemical changes in the brain. However, if the change of one neurotransmitter value is such that other neurotransmitters cannot compensate for its effects, inevitably the system goes into the intermittency mode, and the subject cannot keep his/her attention level, and task switching is observed in his/her behavior. In other words, the system goes out from its self-organizing region, and drug consumption or other treatments can restore the system to its normal mode. Therefore, the performance of the attention controlling system is not only influenced by the absolute values of different neurotransmitter secretions, but also by their relative values.

The proposed model shows that in contrary to popular belief, the dopamine secretion system may work properly, but the interaction between neural systems is impaired so as to cause the person to have difficulty in keeping his/her attention level over time. Thus it can be predicted that in healthy individuals, the attention control system can adaptively set the

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**Fig. 13.** The model behavior with a variation of \((B)\) and \((K)\) for different randomly selected values of \((A)\).
appropriate proportionality between neurotransmitter values. This setting can be done by sending commands to different neurotransmitter secretion systems to change their behavior appropriately. These behavior changes do not mean a change in the inherent characteristic of the components that are involved in the attention mechanism, but also relate to a change in the type of interactions between components that work together to exchange information in the attentional system. However, in people with ADD, these interaction and adaptive settings cannot be performed perfectly.

In summary, the proposed model in this study presents different significant and considerable predictions about the causes of difficulties in sustaining attention over a period of time. However, a closer look at these forecasts requires the design of some appropriate attentional tests and investigations of the brain function during these tests using FMRI technology. The results of these investigations could help to offer new appropriate treatments for such a disorder.

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