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ORIGINAL PAPER

Layla and Majnun: a complex love story

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Abstract Following previous work, this paper introduces a new dynamical model involving two differential equations describing the time variation of behavior displayed by a couple in a romantic relationship. This model is different from previous ones because it uses complex variables. Since complex variables have both magnitude and phase, they are better able to represent love and can represent more complex emotions such as coexisting love and hate. The model treats feelings as a two-dimensional vector rather than a scalar, which is a step closer to reality. Another interesting characteristic of the new model is its ability to show transiently chaotic behavior between only two individuals, which in previous models appeared only in love triangles. The sensitive dependence on initial conditions represents the unpredictable dynamics of love affairs.

Keywords Love · Differential equations · Complex variables

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1 Introduction

There is a growing interest in using mathematics and computational methods in psychology [1–10]. Some mathematicians and psychologists have introduced mathematical models for romantic relationships [3,7,11–28]. A difficulty in love modeling is quantifying love in some meaningful way [27]. In [26], a series of love models was provided describing different kinds of romantic relationships. Fractional-order models [7,29] and time-delay models [14] have been considered as a way to include memory. The goal of most of these works is to illustrate the complexity that can arise in even minimal dynamical models when the equations are nonlinear.

Although systems with complex variables have been introduced in areas such as rotor dynamics [30], plasma physics [31], optical systems [32], and high-energy accelerators [33], and some other areas of science [34–40], the study of chaotic systems with complex variables is a more recent pursuit [41,42].

To our knowledge, there is no work on using complex variables to describe romantic relationships. This paper introduces a new dynamical model involving two differential equations describing the time variation of the feelings displayed by two individuals in a romantic relationship using complex variables for love. Since complex variables have both magnitude and phase, they are capable of representing more types of emotion. For example, the opposite of love may be apathy rather than hate, and two feelings can coexist if one loves some things about one's partner and hates other things. Another interesting characteristic of the new model is its ability to show transiently chaotic behavior between only two individuals, which in previous works appeared only in love triangles. The sensitive dependence on initial conditions leads to a degree of unpredictability which is common in romantic relationships. Our model is inspired by the love affair between Layla and Majnun, a famous Eastern couple. The rest of the paper is organized as follows:

In the next section, we describe the story of Layla and Majnun. We interpret the role of complex variables in the relationship and propose a possible model that is simple and meaningful. Section 3 includes numerical solutions of the resulting equations, and Sect. 4 is the conclusion.

2 Proposed model

2.1 Layla and Majnun story

There are many famous romantic couples in different cultures, such as "Romeo and Juliet" [28], "Samson and Delilah," "Farhad and Shirin," "Khalip and Mylob," "Cyrano de Bergerac" [23], and "Laura and Petrarch" [17,18].

Here we choose "Layla and Majnun." Majnun means "mad" in Persian (although the origin of Majnun is Arabic), and in our model the lovers are madly in love! It is based on the real story of a young man called Majnun who was a poet. Upon seeing Layla, he fell passionately in love with her and sought her hand in marriage. However, he went mad when Layla's father prevented them from marrying and forced Layla to marry someone else. Majnun began wandering the surrounding desert and writing poems about their tragic love story. Layla became ill from what happened and died. Majnun was later found dead in the wilderness.

In one of his famous poems, he says:

"I pass by these walls, the walls of Layla
And I kiss this wall and that wall
It's not Love of the houses that has taken my heart
But of the One who dwells in those houses"

2.2 Interpretation of complex variables

There are at least two ways to interpret complex variables as a description of romantic feelings:

1. The feelings could be a combination of love and hate. The complex variable has a magnitude and a phase between 0° and 180° (π), with the variable and its complex conjugate indistinguishable. The magnitude represents the intensity of romantic emotion, with zero corresponding to complete apathy. The phase represents the degree of love and hate, with zero corresponding to pure love and 180° (π) to pure hate. Any angle between these extremes is a kind of ambivalence in which love and hate coexist.
2. The magnitude of the vector could represent the intensity of feelings, while the two orthogonal components (the real part and imaginary parts) could be any two quantities that do not strongly correlate, such as love and respect.

We prefer the first way, although some may suggest a third way for this interpretation. Complex variables provide a more realistic description of common romantic feelings and allow one to describe the quality of love as well as the quantity.

2.3 Proposed complex model for love

Although models for romantic relationships can be arbitrarily complicated with numerous parameters and degrees of freedom, we consider here one of the simplest possible nonlinear models involving only two complex variables and four parameters in an attempt to show that even that simple model can display interesting and plausible behavior. Our proposed model is as follows:

$$\begin{aligned} \frac{\mathrm{d}M}{\mathrm{d}t} &= a + L^2 + cM \\ \frac{\mathrm{d}L}{\mathrm{d}t} &= b + M^2 + dL \\ a &> 0 \\ b, c, d &< 0 \end{aligned} \tag{1}$$

The complex variables M and L are the respective feelings that Majnun and Layla have for one another. There is a detailed explanation of the meaning of the parameters in different love models and styles in [26]. Here, a and b are constant terms describing the effect of the environment on their love. Majnun was a free man. He was composing poems about their love, and everyone had sympathy for him. Therefore, the overall environmental effect on him was encouraging ($a > 0$). On the other hand, Layla was blamed by her own family (especially her father), and since she was married, most people interpreted her love for Majnun as unfaithfulness. So the overall environmental effect on her was discouraging ($b < 0$). Their love was not an ordinary one. They were madly in love, and any sign of attention from the other encouraged them considerably (the squared terms). Furthermore, their love was a kind of pure love, responding strongly to the feelings of the other but devoid of greed and selfishness ($c, d < 0$). Selecting negative c, d makes the model applicable to a wide range of lovers. It represents a kind of secure love as described in previous models [13,26] which claimed that secure people probably represent the majority of the population. In earlier models, secure individuals cannot have cyclic love dynamics, and their love will be stable. However, in real-world relationships, perfect stability is rare, and cyclic dynamics seem more common. In the next section, we show that complex variables allow our model to represent both stable and cyclic behavior as well as transient chaos.

3 Simulation results

We used MATLAB 7.6.0 (R2008a) for our simulations with the default differential equation solver Ode45, which uses an optimum-adaptive step size.

Selecting $a = 1, b = -1$, we analyze the model using c, d as the bifurcation parameters.

Considering $M = M_r + iM_i$ and $L = L_r + iL_i$, we can rewrite (1) as follows:

$$\begin{aligned}
\frac{\mathrm{d}M_r}{\mathrm{d}t} &= 1 + L_r^2 - L_i^2 + cM_r \\
\frac{\mathrm{d}M_i}{\mathrm{d}t} &= 2L_rL_i + cM_i \\
\frac{\mathrm{d}L_r}{\mathrm{d}t} &= -1 + M_r^2 - M_i^2 + dL_r \\
\frac{\mathrm{d}L_i}{\mathrm{d}t} &= 2M_rM_i + dL_i \\
c, d &< 0
\end{aligned} \tag{2}$$

The equilibria of the above equations are determined by the solution of the following:

$$\begin{aligned}
&1 + L_r^2 - L_i^2 + cM_r = 0 \\
&2L_rL_i + cM_i = 0 \\
&-1 + M_r^2 - M_i^2 + dL_r = 0 \\
&2M_rM_i + dL_i = 0
\end{aligned} \tag{3}$$

Analytical solution of the resulting eighth-order polynomial is difficult. However, if we choose $c = 0$, the eight equilibria are given by

$$\begin{aligned}
M_r &= \pm\frac{d}{2}\frac{1}{\sqrt{\frac{-1+\sqrt{1+d^2}}{2}}} \\
M_i &= \pm\sqrt{\frac{-1+\sqrt{1+d^2}}{2}} \\
L_r &= 0 \\
L_i &= \pm 1
\end{aligned} \tag{4}$$

Although calculating eigenvalues for these equilibrium points is difficult in general, we can find values for some particular choices of the parameters using the Jacobian of Eq. 2 given by

$$J = \begin{vmatrix} c & 0 & 2L_r & -2L_i \\ 0 & c & 2L_i & 2L_r \\ 2M_r & -2M_i & d & 0 \\ 2M_i & 2M_r & 0 & d \end{vmatrix} \tag{5}$$

In what follows, we show three different types of behavior that are typical of the varied dynamics that the model can exhibit.

3.1 Case 1: an ever-after story

For a wide range of parameters, the model exhibits stable solutions. After a transient, the dynamics settle to a fixed point and remain there forever. Depending on the parameters and initial conditions, these fixed points can represent happy or unhappy endings. We select values for the parameters that make it easier to do some parts of the calculation analytically. Selecting $c = 0,\ d = -4\sqrt{5}$ gives the values in Table 1.

Table 1 Equilibria and eigenvalues for $(a, b, c, d) = \left(1, -1, 0, -4\sqrt{5}\right)$ that give a stable solution

Equilibria (M_r, M_i, L_r, L_i)	Eigenvalues
$\left(+\sqrt{5}, +2, 0, +1\right)$ $\left(+\sqrt{5}, -2, 0, -1\right)$	$\lambda_{1,2} = -0.8002 \pm 1.2179\,i$, $\lambda_{3,4} = -8.1441 \pm 1.2179\,i$
$\left(-\sqrt{5}, +2, 0, -1\right)$ $\left(-\sqrt{5}, -2, 0, +1\right)$	$\lambda_{1,2} = 0.8848 \pm 0.8348\,i$, $\lambda_{3,4} = -9.8291 \pm 0.8348\,i$

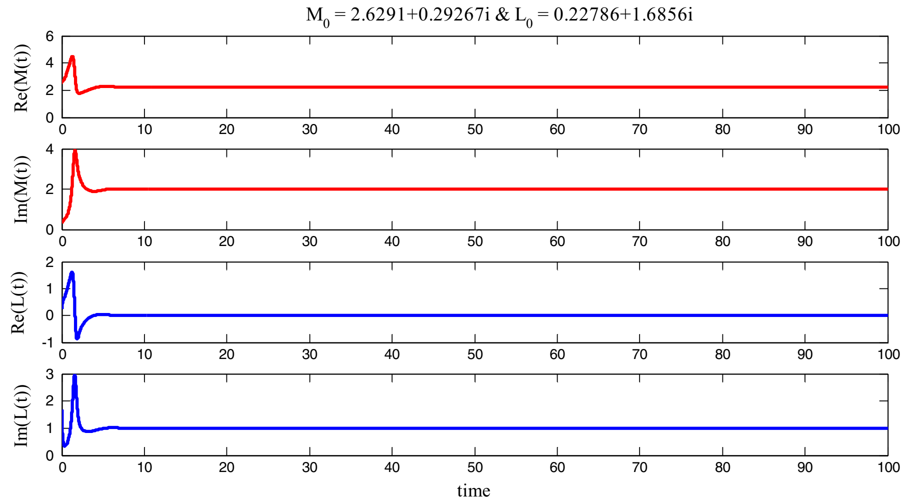

Fig. 1 Simulation results of Eq. 1 with $a = 1$, $b = -1$, $c = 0$, $d = -4\sqrt{5}$ for 100 time units. The initial conditions are indicated in the figure. As can be seen, this relationship converges to a stable situation after a short transient. This situation is proper for ever-after stories and doesn't sound realistic in many cases

Table 2 Equilibria and eigenvalues for $(a, b, c, d) = \left(1, -1, 0, -\sqrt{3}\right)$ that give a stable limit cycle

Equilibria (M_r, M_i, L_r, L_i)	Eigenvalues
$\left(+\frac{\sqrt{6}}{2}, +\frac{\sqrt{2}}{2}, 0, +1\right)$ $\left(+\frac{\sqrt{6}}{2}, -\frac{\sqrt{2}}{2}, 0, -1\right)$	$\lambda_{1,2} = 0.4074 \pm 1.9235\,i$, $\lambda_{3,4} = -2.1394 \pm 1.9235\,i$
$\left(-\frac{\sqrt{6}}{2}, +\frac{\sqrt{2}}{2}, 0, -1\right)$ $\left(-\frac{\sqrt{6}}{2}, -\frac{\sqrt{2}}{2}, 0, +1\right)$	$\lambda_{1,2} = 1.3300 \pm 1.1154\,i$, $\lambda_{3,4} = -3.0621 \pm 1.1154\,i$

In this case, at least one of the equilibria is a stable focus, and the expected dynamic is thus a damped oscillation. Figure 1 shows the result of a numerical simulation of Eq. 1 with $c = 0$, $d = -4\sqrt{5}$. The system converges to the considered equilibrium as expected.

3.2 Case 2: vertigo

Here, we try to find cyclic solutions. Although such solutions may seem more dynamic and thus more real-

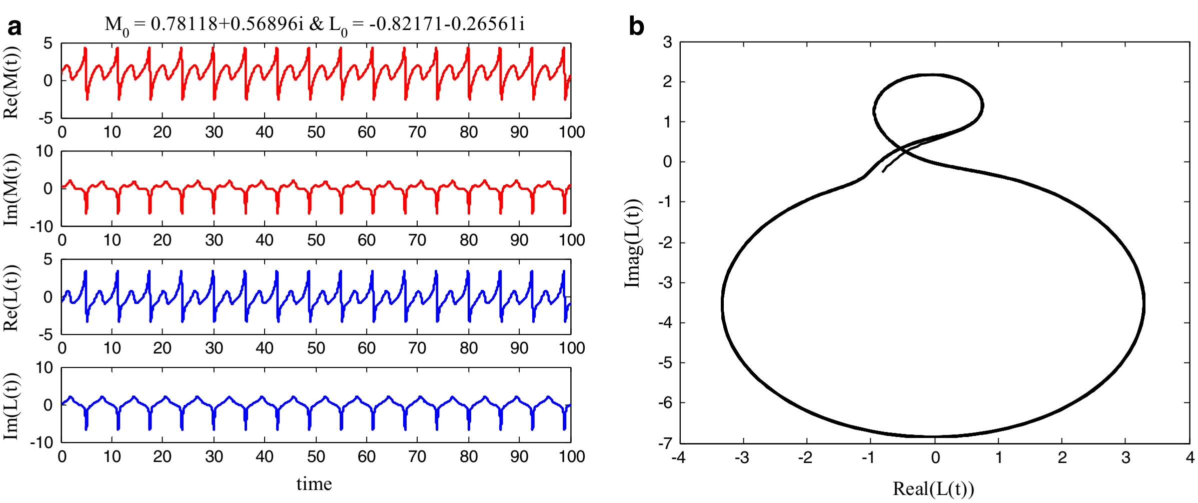

Fig. 2 Simulation results of Eq. 1 with $a = 1$, $b = -1$, $c = 0$, $d = -\sqrt{3}$ for 100 time units. The initial conditions are indicated in the figure. **a** Time series, **b** 2D trajectory of Layla's love in the complex plane. As can be seen, this relationship converges to a periodic behavior after a short transient. This situation is very common in real life and most of the existing models for romantic relationships present such dynamic

Table 3 Equilibria and eigenvalues for $(a, b, c, d) = (1, -1, 0, 0)$ that give transient chaos

Equilibria (M_r, M_i, L_r, L_i)	Eigenvalues
$(\pm 1, 0, 0, \pm 1)$	$\lambda_{1,2,3,4} = \pm\sqrt{2} \pm \sqrt{2}i$

istic, they are too predictable. Selecting $c = 0$, $d = -\sqrt{3}$ gives the values in Table 2.

Figure 2 shows the result of solving Eq. 1 with $c = 0$, $d = -\sqrt{3}$. As can be seen, the dynamic is a stable limit cycle.

3.3 Case 3: a little madness

Although the equations apparently do not admit chaotic attractors, there are solutions that are transiently chaotic, especially when the damping terms c and d are small or zero. We know that life is sensitive and unpredictable, especially when one falls in love. There are occasional illogical, sudden, and explosive events when two people are madly in love. Selecting $c = d = 0$ gives the values in Table 3.

Figures 3, 4, and 5 show the result of solving Eq. 1 with $c = d = 0$ for different initial conditions. The time series are transiently chaotic with sudden occasional spikes. For example, although there is a transiently chaotic behavior in the time series shown in Fig. 3, if we continue that simulation for a longer time, there would be an unbounded explosion near 800 time units (Fig. 4).

This case is special since it is a conservative Hamiltonian system with a Hamiltonian given by $H = L + \frac{L^3}{3} + M - \frac{M^3}{3}$. Thus, it has no attractor, and every initial condition potentially leads to a unique behavior. While we know that initial impressions count in romance, this is a case where they are never forgotten. Figure 6 shows the calculated H for the last case. As can be seen, the value of H remains nearly constant confirming the accuracy of the calculation.

As can be seen, there are different types of behavior depending on the parameters c and d. There are stable equilibria, stable limit cycles, and transiently chaotic solutions. In transiently chaotic ones, the chaotic sea apparently stretches to infinity, but the trajectory usually returns quickly. Although this type of explosive behavior may not be common for most people, we note that legendary lovers are not at all ordinary. They are not very stable, and many of their actions seem illogical to others. They are very sensitive, and emotional eruptions frequently occur. In fact, among the three possible behaviors, transient chaos is the one best matching Layla and Majnun.

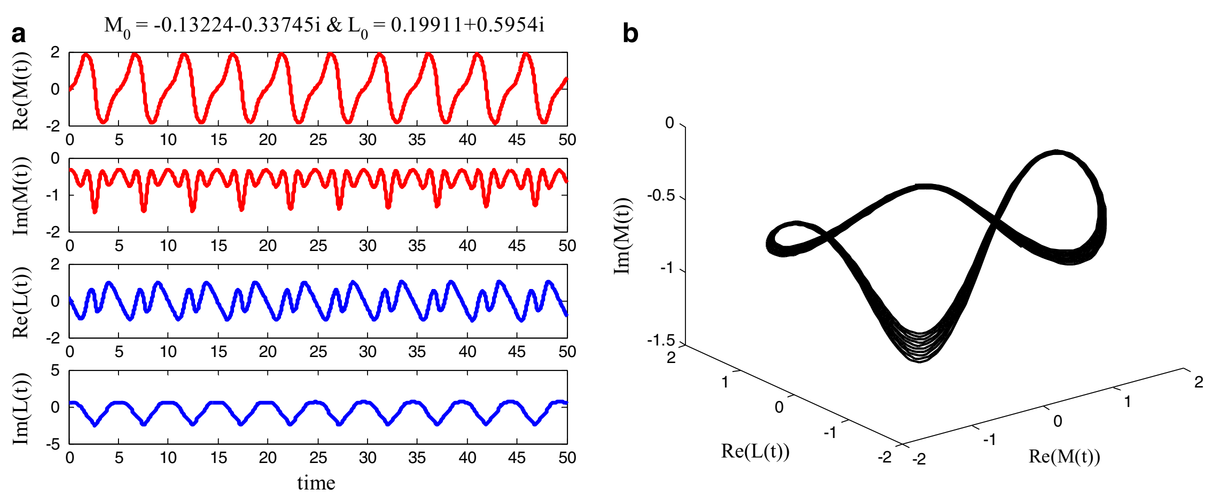

Fig. 3 Simulation results of Eq. 1 with $a = 1$, $b = -1$, $c = d = 0$ for 50 time units. The initial conditions are indicated in the figure. **a** Time series, **b** 3D trajectory in phase space (due to visualization limits, Im(L(t)) has not been shown). As can be seen, this relationship is transiently chaotic. This part of the trajectory is periodic-like (but not exactly periodic) which is much more realistic than the pure periodic behavior

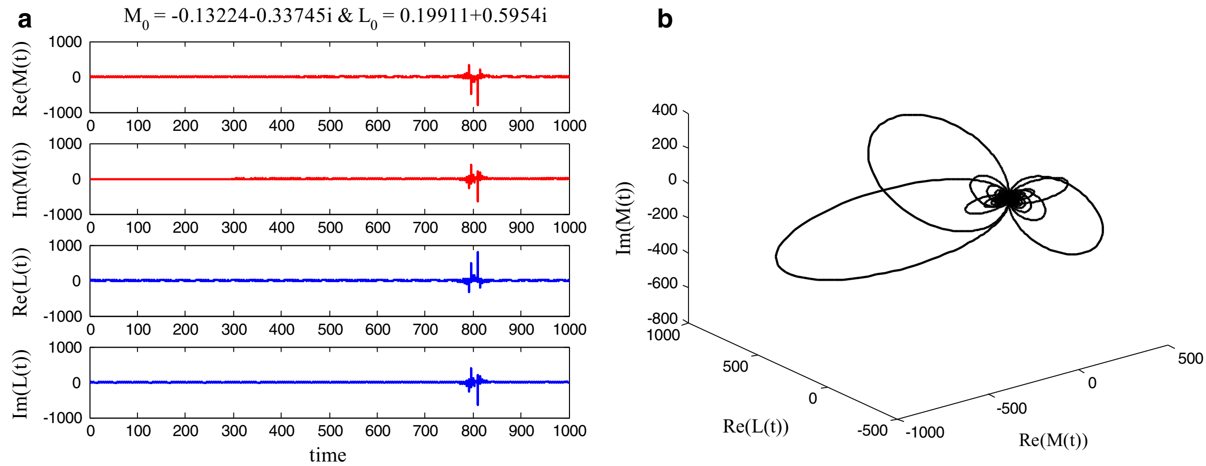

Fig. 4 Simulation results of Eq. 1 with $a = 1$, $b = -1$, $c = d = 0$ for 1000 time units. The initial conditions are indicated in the figure. **a** Time series, **b** 3D trajectory in phase space (due to visualization limits, $\text{Im}(L(t))$ has not been shown). As can be seen, this relationship is transiently chaotic which contains parts with sudden explosion which can be interpreted as a kind of madness (this behavior is rare, but certainly exists in real world)

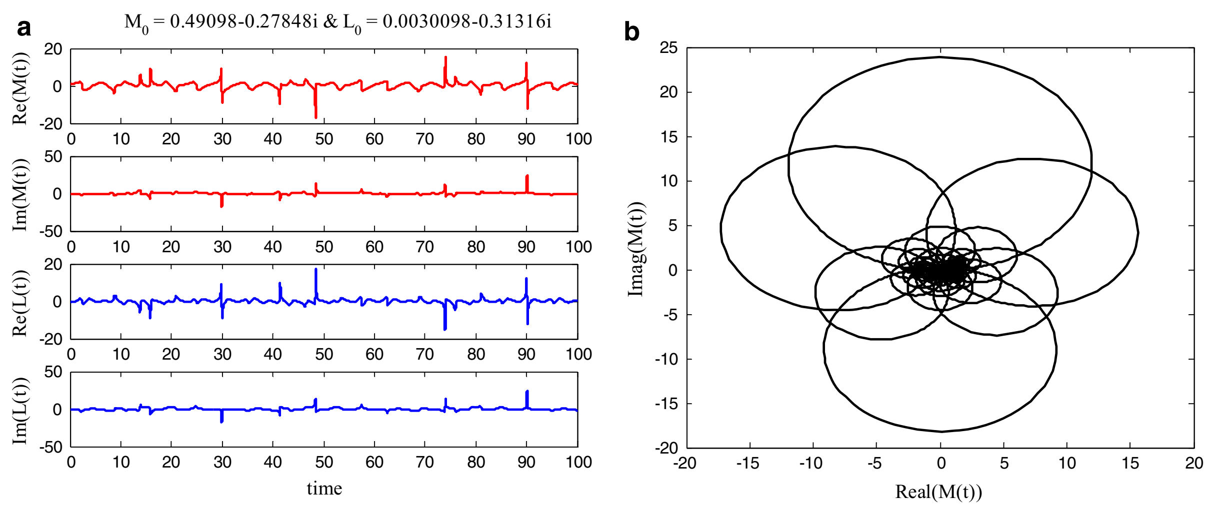

Fig. 5 Simulation results of Eq. 1 with $a = 1$, $b = -1$, $c = d = 0$ for 100 time units. The initial conditions are indicated in the figure. **a** Time series, **b** 2D trajectory of Majnun's love in the complex plane. As can be seen, this relationship is transiently chaotic without those sudden explosions which is very common in reality

3.4 Sensitive dependence on initial conditions

The signature of transient chaos is the sensitive dependence on initial conditions, and this property is easily demonstrated. Figure 7 shows the time series of L_r from simulation of Eq. 1 with $a = 1, b = -1, c = -0.01, d = -0.01$ for a given initial condition and one perturbed by 1 %. It is evident that the system shows sensitive dependence on initial conditions.

4 Conclusion

Complex variables are more realistic for describing phenomena in which quality has an unavoidable role. Using them enables one to represent reality more closely since vectors are more realistic than scalars. When we model romantic relationships with differential equations, we prefer models that show complex and uncertain behaviors (as in the real world) while remain-

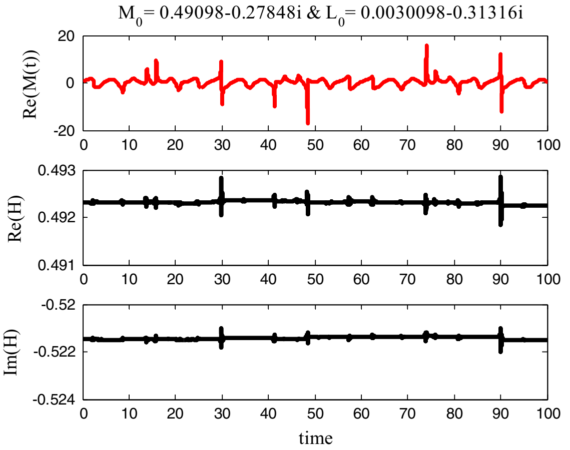

Fig. 6 Calculation of $H = L + \frac{L^3}{3} + M - \frac{M^3}{3}$ with $a = 1,\ b = -1,\ c = d = 0$ for 100 time units

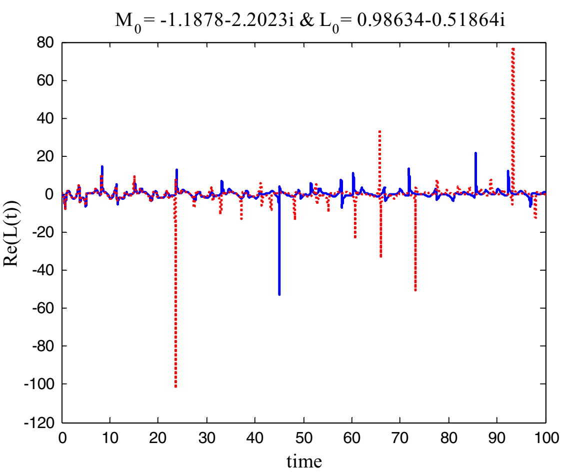

Fig. 7 Evolution of Layla's feelings for Majnun from Eq. 1 with $a = 1,\ b = -1,\ c = -0.01,\ d = -0.01$ for 100 time units showing the effect of changing the initial conditions by 1 %

ing algebraically simple. The model introduced here is simple, meaningful, and capable of showing a variety of different behaviors even with changing only one control parameter. As a suggestion for future works, some can consider the effect of noise (which certainly exists in real situation) on the states and parameters of the proposed model.

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