**Automatic Generation of Fractal Art**

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**Introduction**

The recent realization that very simple equations can have extremely complicated graphical solutions has surprised most scientists and delighted many artists. Inexpensive computers and sophisticated software, now widely available, are powerful new tools for both the scientist and the artist. The difficulty is that most equations produce mundane solutions, and so one usually resorts to extensive experimentation or the experience of others to find interesting cases. An alternative is to program the computer to explore a vast range of equations at random and select those that produce interesting patterns. I will review my attempts over the past decade to automate this process and produce appealing images with minimal human interaction. Whether patterns produced entirely by a computer can be considered "art" is a philosophical debate best left to others.

**Strange Attractors**

Most processes in nature can be described by deterministic equations that uniquely predict the future based on the present. Whether it is the trajectory of a spacecraft crossing the Solar System or the motion of air in a tropical storm, or the spread of an epidemic, what happens next is determined by current conditions. However, it may be that a small change in the present leads to a greater change tomorrow and an even greater change the day after, until eventually all predictability is lost. Equations with this property are said to exhibit "chaos". The graph of a chaotic process is a special kind of fractal called a "strange attractor". A fractal is a geometrical form with infinitely many replicas of itself embedded on ever smaller scales. A strange attractor is a fractal produced by a chaotic dynamical system. Hence, detecting chaos is a good starting point for identifying equations capable of producing visually interesting patterns.

If you were to write down a hundred arbitrary equations, only a couple of them would have chaotic solutions. However, the computer can easily test for chaos by solving the equations for two different starting conditions to see if the solutions rapidly diverge from one another. The Lyapunov exponent is a measure of the divergence, and a positive value signifies chaos. Chaotic cases produce strange attractors; each of which, like snowflakes, is unique and usually beautiful. Even the same equations can produce very different patterns depending on the values of terms in the equation. Figure 1 shows a typical example.

To make an image of this type requires two equations, one for the horizontal position and the other for the vertical position of each successive dot that makes up the image. These coupled equations are iterated repeatedly to produce a sequence of arbitrarily many points. After a while, most of the dots fall on top of previous ones (the attractor), and the calculation can terminate.

![Figure 1: Typical 2-D strange attractor.](image1)

![Figure 2: Typical 3-D strange attractor with the third dimension rendered in grayscale.](image2)

Not all strange attractors produced in this way are equally interesting. Some are too thin or consist of relatively few isolated dots. Others are too thick and eventually fill the screen solidly. This characteristic can be quantified by calculating the fractal dimension, which for an attractor of this type can take on any value between zero (a collection of isolated dots) and two (a surface), and it is usually not an integer.

Experiments with human subjects indicate that most people prefer attractors with dimension about 1.2 and with small positive Lyapunov exponent. Thus, the computer can be instructed to discard cases that do not satisfy these conditions. The dimension preference is reasonable since many natural objects such as rivers and tree branches have dimensions in this range. The Lyapunov exponent preference is more mysterious since it's a dynamical rather than a geometrical property, but it suggests that humans favor some unpredictable, but not too much. Attempts to discern individual differences in preferences between artists and scientists lead to mixed results.
Strange Attractors can be produced in dimensions higher than 2, but that requires additional means of visualization. One possible method is to code the third dimension in color. An example of an attractor produced in this way is shown in Figure 2.

The third (or higher) dimension can also be coded into a brightness scale, or the attractor can be viewed as an anaglyph (using red-blue glasses), stereogram, or stereo pair viewed with crossed eyes. The attractor can also rotate in an animated view.

Iterated Function Systems

Another means of producing fractal patterns uses iterated function systems (IFS). An IFS is a dynamical system with no (or more) rules that tell where to go next based on the current position. The rules are chosen randomly such as by flipping a coin or using the computer random number generator. Although the sequence of points clearly depends on the articular random choices, surprisingly, the final pattern does not. A typical image produced by this method is shown in Figure 3.

An IFS pattern can also be classified according to its fractal dimension and Lyapunov exponent. As with strange attractors, human subjects seem to prefer images with a fractal dimension between 1 and 2, but within a wider range. Although the sequence is determined randomly, the same sequence will cause two initial points to converge to the same sequence of values. Therefore, these systems have negative Lyapunov exponents with a maximum value that depends on the fractal dimension. Human subjects prefer cases with the largest negative exponents. Thus, the computer can be programmed to select cases that are likely to be visually appealing.

Generalized Julia Sets

Some of the most intricate fractals are produced by Julia sets and their cousins. These systems are also dynamical, but the points that evolve with successive iteration are not plotted. Rather, one plots at each initial position a color determined by the number of iterations required for its orbit to escape beyond some specified region or by some other criterion. As with the previous cases, most choices of equations lead to uninteresting results. The interesting cases are those for which the orbit escapes, but only slowly. Thus, the computer can discard cases for which an orbit that starts near the center of the image escapes in less than 100 iterations or more than about 1000 iterations. The remaining images are typically appealing. A sample image produced by this method is shown in Figure 4.

Symmetric Icons

A problem with many of the images produced by the above methods is that they are too unstructured. A simple way to add structure is to take each image and distort it in some fashion so that it occupies only a portion of the plane on which it is displayed. It is then replicated in other portions of the plane, perhaps with rotations and reflections to produce a symmetric icon. Almost any fractal that meets the conditions above looks even more interesting when displayed in this way. Figure 5 shows a typical example.

Other transformations can also be performed on the image. For example, it can be wrapped onto a cylinder, sphere, or torus and then projected back onto the plane.

Artificial Neural Networks

Artificial neural networks are computer models that attempt to emulate the structure and operation of the brain. They consist of a large number of artificial neurons that take their input as the sum of the output of other neurons and produce an output that is some nonlinear function of the input.
Normally, they are trained to perform some operation on data supplied at the input and generate an appropriate output response. However, if the output is fed back to the input, the network behaves as a dynamical system and produces a sequence of output values that may be chaotic. With random connection strengths between the neurons, such networks can be used to produce visual patterns using one neuror or combination of neurons to control the horizontal position, one the vertical position, and a third the color of each point. They produce interesting patterns similar to the strange attractors above, except that they are usually constructed so that all orbits are bounded to some predetermined region of space. Consequently, many more cases are chaotic, and there is less need to discard cases. Figure 6 shows a typical such attractor.

These cases are selected automatically by simply counting the pixels illuminated on the screen as the pattern develops and discarding those with too few or too many such pixels.

Neural networks are usually designed to facilitate training. Thus, the connection strengths of the network can be adjusted to produce visually interesting patterns after training on a set of images that have been aesthetically evaluated by a human. I am currently working on this prospect and routinely use a trained neural network to prescreen the fractals of the day that have appeared in my fractal gallery on the Web (http://sprott.physics.wisc.edu/fractals.htm) since 1996. The technique shows promise but needs additional development. As computers become more powerful and software more sophisticated, the time may come when such programs rival humans in the quality of the images that they are capable of creating.

Rihlingraphy