Exam 2

Print your name and section clearly on all five pages. (If you do not know your section number, write your TA's name.) Show all work in the space immediately below each problem. Your final answer must be placed in the box provided. Problems will be graded on reasoning and intermediate steps as well as on the final answer. Be sure to include units wherever necessary, and the direction of vectors. Each problem is worth 25 points. In doing the problems, try to be neat. Check your answers to see that they have the correct dimensions (units) and are the right order of magnitude. You are allowed one 8½ x 11” sheet of notes and no other references. The exam lasts exactly 50 minutes.

(Do not write below)

SCORE:

Problem 1: ALMQVIST

Problem 2: FEELEY

Problem 3: FANTER

Problem 4: KRUGER

TOTAL: ___________
1. A solenoid with a length of 30 cm and a diameter of 2 cm has 100 turns of wire distributed uniformly along its length and produces a magnetic field of 20 Gauss in its interior.

   a. What is the current in the wire? (5 pts.)

   \[ \mathbf{B} = \mu_0 \mathbf{N} \mathbf{I} / \ell \]

   \[ I = \frac{\mathbf{B} \ell}{\mu_0 \mathbf{N}} = \frac{(0.002)(0.3)}{4\pi \times 10^{-7} \times 100} = 4.77 \text{ A} \]

   b. What is the magnetic flux in the solenoid? (5 pts.)

   \[ \Phi = \mathbf{B} \mathbf{A} = \mathbf{B} \pi r^2 \]

   \[ = 0.002 \pi (0.01)^2 = 6.28 \times 10^{-7} \text{ Wb} \]

   c. What is the inductance of the solenoid? (5 pts.)

   \[ L = \frac{N \Phi}{I} = \frac{100 \times 6.28 \times 10^{-7}}{4.77} \]

   \[ = 1.32 \times 10^{-5} \text{ H} \]

   d. What is the total magnetic energy stored in the solenoid? (5 pts.)

   \[ U = \frac{1}{2} L I^2 \]

   \[ = 0.5 \times 1.32 \times 10^{-5} (4.77)^2 \]

   \[ = 1.50 \times 10^{-4} \text{ J} \]

   e. What maximum velocity can an electron \((m_e = 9.1 \times 10^{-31} \text{ kg})\) have and be confined within the solenoid? (5 pts.)

   \[ n = \frac{m \nu}{q \Phi} \]

   \[ \nu = \frac{q \Phi}{m} \]

   \[ = \frac{1.6 \times 10^{-19} \times 0.002 \times 0.01}{9.1 \times 10^{-31}} = 3.52 \times 10^6 \text{ m/s} \]
2. A single-turn circular loop of wire with a radius of 10 cm rotates 60 times per second with its axis perpendicular to a magnetic field of 0.5 T.

   a. Calculate the peak voltage induced in the loop. \(10 \text{ pts.}\)

   \[
   V = \frac{d\Phi}{dt} = BA\omega \sin \omega t
   \]

   \[
   V_0 = BA\omega = B\pi r^2 2\pi f
   \]

   \[
   = 0.5 \pi r^2 \times 0.01 \times 120
   \]

   \[
   = 5,92
   \]

   \[
   5.92 \text{ V}
   \]

   b. Calculate the rms voltage induced in the loop. \(5 \text{ pts.}\)

   \[
   V_{\text{rms}} = V_0 / \sqrt{2}
   \]

   \[
   = 4.19
   \]

   \[
   4.19 \text{ V}
   \]

   c. If the loop has a resistance of 2 \(\Omega\), what average power is dissipated in the loop? \(10 \text{ pts.}\)

   \[
   \bar{P} = \frac{V_{\text{rms}}^2}{R} = \frac{(4.19)^2}{2}
   \]

   \[
   = 8.77
   \]

   \[
   8.77 \text{ W}
   \]
3. After being closed for a long time, the switch in the circuit below is opened at \( t = 0 \).

\[
\begin{align*}
\text{+} & \quad \text{+} \\
- & \quad \text{+} \\
9 \text{ V} & \quad 1 \Omega \\
- & \quad 2 \Omega \\
\text{+} & \quad \text{+}
\end{align*}
\]

3 H

a. What is the current in the 1-\( \Omega \) resistor just before the switch opens? (5 pts.)

\[
I_1 = \frac{V_o}{R_1} = \frac{9}{1} = 9
\]

[9 A]

b. What is the current in the 2-\( \Omega \) resistor just before the switch opens? (5 pts.)

\[
I_2 = \frac{V_2}{R_2} = \frac{0}{2} = 0
\]

[0 A]

c. What is the current in the inductor just after the switch opens? (5 pts.)

\[
I_L = \frac{V_o}{R_1} = \frac{9}{1} = 9
\]

[9 A]

d. What is the voltage across the inductor just after the switch opens? (5 pts.)

\[
V_L = V_2 = I_2 R_2 = I_L R_2
\]

\[
= 9 \times 2 = 18
\]

[18 V]

e. What is the voltage across the inductor at \( t = 0.5 \text{ s} \)? (5 pts.)

\[
V = V_0 e^{-\frac{R t}{L}}
\]

\[
= 18 e^{-2 \times 0.5 / 3}
\]

\[
= 12.9
\]
4. In the circuit below, the voltage source is sinusoidal.

\[ 9 \sin 400t \]

\[ 40 \Omega \]

\[ 0.5 \text{ H} \]

\[ 10 \mu \text{F} \]

a. Calculate the peak value of the current \( i \). (15 pts.)

\[ I_0 = \frac{V_o}{Z} = \frac{V_o}{\sqrt{R^2 + (X_L - X_C)^2}} \]

\[ X_L = \omega L = 400 \times 0.5 = 200 \Omega \]

\[ X_C = \frac{1}{\omega C} = \frac{10^6}{400 \times 10} = 250 \Omega \]

\[ I_0 = \frac{9}{\sqrt{40^2 + 50^2}} = 0.141 \]

\[ 141 \text{ mA} \]

b. Calculate the phase of \( i \) relative to the source. (10 pts.)

\[ \phi = \tan^{-1} \left( \frac{X_L - X_C}{R} \right) \]

\[ = \tan^{-1} \left( -\frac{50}{40} \right) \]

\[ = -51.3^\circ \]

\[ X_C > X_L \Rightarrow \text{Current leads voltage} \]

\[ 51.3^\circ \]