A Tribute to J. C. Sprott

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In honor of his 75th birthday, we review the prominent works of Professor Julien Clinton Sprott  
in chaos and nonlinear dynamics. We categorize his works into three important groups. The first  
and most important group is identifying new dynamical systems with special properties. He  
has proposed different chaotic maps, flows, complex variable systems, nonautonomous systems,  
partial differential equations, fractional-order systems, delay differential systems, spatiotemporal  
systems, artificial neural networks, and chaotic electrical circuits. He has also studied dynamical  
properties of complex systems such as bifurcations and basins of attraction. He has done work  
on generating fractal art. He has examined models of real-world systems that exhibit chaos. The  
second group of his works comprise control and synchronization of chaos. Finally, the third group  
is extracting dynamical properties of systems using time-series analysis. This paper highlights  
the impact of Sprott’s work on the promotion of nonlinear dynamics.

Keywords: Chaos; dynamical system; dynamical property; control; synchronization.

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1. Introduction

JULIEN CLINTON SPROTT earned his bachelor’s degree from MIT in 1964 and his PhD in physics from the University of Wisconsin in 1969. He is an Emeritus Professor of Physics at the University of Wisconsin-Madison. He is the author or coauthor of about 500 papers and a dozen books on topics including plasma physics, chaos, dynamical systems, and fractals. After a 25-year career in experimental plasma physics, he became interested in chaos in 1988 when George Rowlands from the University of Warwick gave a colloquium on the topic for the Physics Department at his university.

Using the primitive personal computers of the day, he developed programs to automate the search for new chaotic systems in an attempt to quantify how common are chaotic solutions in various types of dynamical systems [Sprott 1993a, 1993b]. That work led to a search for the algebraically simplest examples of chaotic systems of various types and to the identification of many new chaotic systems with special properties, including many that are algebraically simpler than any that were previously known [Sprott 1993b, 1997a, 1997b, Linz & Sprott 1994, Sprott 2008d]. His search and analysis programs have been continually refined and widely used for many important subsequent studies.

Professor Sprott has been especially helpful to students and junior scientists in developing their ideas and getting them published. He enjoys his collaborations with such people and humbly admits that most of the ideas for his many recent publications came from his coauthors. The rest of this paper will discuss this research and its importance.

2. Review of Publications

Nonlinear dynamics is an important area of research that has received great attention in recent years, especially since the advent of inexpensive and powerful personal computers. Chaotic behavior is one of the most important phenomena in nonlinear systems. Structural properties of systems that are capable of generating chaotic attractors are a challenging subject in this area. For example, hidden and self-excited attractors have been defined and studied by Leonov and Kuznetsov [Kuznetsov et al., 2013b, Kuznetsov et al., 2014, Leonov & Kuznetsov, 2013a, 2014a, 2014b, Leonov et al., 2015, 2014, Leonov & Kuznetsov, 2013a, 2013b, Bogus et al., 2011]. Self-excited attractors are ones whose basin of attraction is associated with an unstable equilibrium, while hidden attractors have basins of attraction that are not associated with any equilibria. Another example is time-reversible chaotic flows in which the governing equations of the system do not change under the transformation $t \rightarrow -t$ except for changing the sign of one or more of the variables [Sprott 2015a]. Continuous-time systems with dimensions greater than three can exhibit hyperchaos in which two or more of the Lyapunov exponents are positive.

Sprott has classified strange attractors into three principal classes, hyperbolic, Lorenz-type, and quasi-attractors and discussed the possibility of finding a rigorous mathematical model describing the chaotic attractors [Elhadj & Sprott 2015b, 2015c]. In 1993 he investigated a subset of maps and flows to determine what fraction of them had chaotic solutions [Sprott 1993a], and estimated the most probable dimension of their strange attractors [Sprott 1994d]. He also showed how chaos evolves from being relatively rare in low-dimensional systems to generic in high-dimensional artificial neural networks [Dechert et al., 1999], and its persistence (smooth variation of Lyapunov exponents with parameter changes) [Albers et al., 2004, Elhadj & Sprott, 2014].

He described his methods for efficiently searching large classes of discrete and continuous-time systems to find those often tiny regions of parameter space that admit chaos [Sprott 1993a, 1993b]. Partial differential equations (PDEs) and systems of many ordinary differential equations as well as discrete-time neural networks can also generate high-dimensional chaos [Sprott 2008d]. He identified those parameters for various common chaotic systems that give the most complex chaotic attractors (the largest attractor dimension) [Sprott 2007a].

Social networks in the real world show weak chaos, and networks of ODEs can model them [Chlouverakis & Sprott, 2008]. Because of the large number of new chaotic systems being published, he proposed a standard for the publication of new chaotic systems that is often used by reviewers in evaluating submissions [Sprott, 2011].

Sprott discussed the concept of robust chaos (the absence of periodic windows or coexisting attractors) [Elhadi & Sprott, 2008d] and proposed a method for generating robust chaos in 2D maps [Zaremba & Sprott, 2013]. He showed that increasing the dimension of the typical dynamical
system leads to increasing the number of positive Lyapunov exponents and decreasing the number of periodic windows [Albers et al. 2006; Albers & Sprott 2006d]. He determined that bifurcations from fixed points of large dynamical systems are typically due to complex eigenvalues leading to a Neimark–Sacker bifurcation [Albers & Sprott 2006c]. He has reviewed 2D rational mappings, hyperbolic structure, structural stability, some questions about periodic, homoclinic, and heteroclinic orbits, classification of chaos, and some questions about chaotification of dynamical systems [Elhadi & Sprott 2011f]. He has studied structural stability as a condition for robustness of invariant sets in 3D quadratic mappings [Zeraoulia & Sprott 2014]. He has shown the existence of invariant sets (universal basins of attraction) for typical nonlinear, high-dimensional dynamical systems [Elhadi & Sprott 2011h]. He has also shown that special nonlinearities with partial information in the variable can lead to chaos (by omitting amplitude or polarity information in the feedback loop) [Li et al. 2016e]. In the following subsections, various aspects of his studies are described in more detail.

### 2.1. Dynamical systems and their properties

This subsection surveys some of Sprott’s work in generating new chaotic systems and investigating their behavior.

#### 2.1.1. Chaotic maps

Some of the simplest examples of chaos arise in iterated maps in which time advances in discrete steps. Sprott has found some new maps with specific fractal attractors [Elhadi & Sprott 2008a, 2010a]. A 2D multifold map with a period-doubling route to chaos has been proposed [Elhadi & Sprott 2008a] as well as an unusual map with a rational function [Elhadi & Sprott 2011l]. Piecewise-linear maps are important because they sometimes facilitate analytic analysis [Elhadi & Sprott 2011l]. A robust homoclinic chaotic system involving a piecewise-smooth map which is a combination of the Hénon and Lozi systems has been discussed [Elhadi & Sprott 2012b] as well as a piecewise-linear map with the same nonlinearity as Chua’s circuit and capable of generating single and multiscroll chaotic and hyperchaotic attractors [Elhadi & Sprott 2009a].

Sprott proposed a criteria for 2D quadratic maps that can ensure the solution is either chaotic or nonchaotic [Elhadi & Sprott 2009a]. The dynamics of a 2D Hénon-like mapping with an unknown bounded function has been discussed [Elhadi & Sprott 2012d].

### 2.1.2. Chaotic flows

Sprott has extensively studied chaotic systems of ODEs [Sprott 2010a]. In 1994, he published a classic paper describing 19 chaotic flows that were simpler than any previously known with only five terms and two quadratic nonlinearities or six terms and one quadratic nonlinearity [Sprott 1994f]. In 1996, he identified the algebraically simplest dissipative chaotic flow with five terms and a single quadratic nonlinearity [Sprott 1997f]. Some other simple chaotic jerk systems with quadratic, cubic, and exponential nonlinearities were introduced [Sprott 1997f, Muneer et al. 2013]. He also found the simplest 3D chaotic system with only one nonlinearity in the form of the modulus of the dynamical variable [Linz & Sprott 1998, Sprott 2006d].

A unified chaotic system containing Lorenz, Chua, and some other systems was introduced [Elhadi & Sprott 2010a]. Hyperjerk systems have been investigated with their chaotic and hyperchaotic attractors [Chlouverakis & Sprott 2006]. A new category of 3D diffeomorphisms with constant Jacobian and different behaviors was proposed [Elhadi & Sprott 2009a]. Sprott has investigated the time evolution of some maps and flows in terms of higher-order moments [Rowlands & Sprott, 2008]. A chaotic jerk system with a piecewise-exponential nonlinearity has been introduced [Sun & Sprott 2008a]. Sprott investigated the equivalence between 4D autonomous dynamical systems and hyperjerk systems [Elhadi & Sprott 2011c]. Some conditions for the nonexistence of homoclinic and heteroclinic orbits and Shilnikov chaos in n-dimensional smooth flows have been...
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presented [Elhadi & Sprott, 2012] and a method for generating \( n \)-scroll chaotic attractors in 3D flows was proposed [Elhadi & Sprott, 2013c].

Sprott also proposed different types of systems with hidden attractors such as systems without any equilibria [Jafari et al., 2013; Safari et al., 2013], systems with one stable equilibrium [Menke et al., 2013; Sprott et al., 2013; Baghani et al., 2013; Safari et al., 2013], systems with lines of equilibria [Jafari et al., 2013; Safari & Sprott, 2014], systems with a plane of equilibria [Jafari et al., 2016], systems with surfaces of equilibria [Jafari et al., 2016], systems with a curve of equilibria [Barati et al., 2016], and systems with circular and square equilibria [Gottthans et al., 2016]. Dynamical behaviors of the Moffatt system and its hidden chaotic attractors have been demonstrated [Wei et al., 2017].

A 4D system without any equilibria with multistability has been investigated [Li & Sprott, 2014]. He has proposed a four-wing chaotic system with symmetric bistability [Li et al., 2015a]. He showed that the Wang-Chen system which has a chaotic attractor with one stable equilibrium has a coexisting limit cycle, giving a system with three types of coexisting attractors [Sprott et al., 2013]. A new chaotic system with multistability was proposed [Li et al., 2015a], and amplitude and phase control methods are applied to it. A 3D flow with involutional symmetry has been presented [Sprott, 2014du] as well as a new 3D chaotic flow with many regions of multistability [Li & Sprott, 2013].

Sprott has proposed two autonomous 3D chaotic flows with a continuously adjustable attractor dimension [Munmuangsaen et al., 2013]. He proposed a 3D chaotic flow, all of whose terms except one are quadratic [Li & Sprott, 2014]. He showed that the extreme multistability which is discussed in [Hooke et al., 2012] can be achieved in any dynamical system by adding extraneous variables and using their initial conditions in place of the existing parameters or as additional parameters [Sprott & Li, 2014].

He described a new system with a hidden attractor and proposed a likelihood score in a Gaussian Mixture Model which can estimate the system’s parameter [Luo et al., 2014]. He proposed a hyperchaotic system with a line of equilibria [Li et al., 2014]. It has two symmetric limit cycles that merge together to form a strange attractor. A simple 3D time-reversible system of ODEs has been described which has nested conservative quasi-periodic tori coexisting with a dissipative chaotic attractor [Sprott, 2014].

A piecewise-linear hyperchaotic system with a signum nonlinearity and absolute values has been presented [Li et al., 2014]. A simple time-reversible 3D chaotic flow with a polynomial nonlinearity has been described [Sprott, 2015]. Some 3D chaotic flows with hidden attractors and multistability have been proposed [Sprott, 2015]. Using a chaotic hyperbolic system and a synchronization technique, he has proposed a method for generating chaotic hyperbolic systems [Elhadi & Sprott, 2012], and he introduced a 3D chaotic jerk system with a single nonhyperbolic equilibrium [Wei et al., 2015].

Sprott showed that perpetual points are limited in finding dissipativity of systems [Jafari et al., 2015b], as proposed by [Prasad, 2015]. Four categories of chaotic flows with and without fixed points and perpetual points have been discussed [Nazarimehr et al., 2015a]. It was shown that perpetual points are not sufficient for locating hidden attractors as originally hoped [Nazarimehr et al., 2015a]. Different behaviors of the Kingni–Jafari system have been investigated [Wei et al., 2016], especially its unstable behaviors.

He explored a new regime of chaotic flows with offset boosting [Li & Sprott, 2014] and a 4D hyperjerk system with an exponential nonlinearity and its realization with field programmable analog arrays [Dakhiran & Sprott, 2014]. Flows with spatially-periodic functions such as the sine can generate elegant chaotic behaviors and deterministic fractional Brownian motion [Sprott & Chlouverakis, 2007]. He proposed a chaotic system with self-reproduction using a sine function that leads to an infinite number of multistable attractors that can be used as a model for memory [Li et al., 2017].

Asymmetric dynamical systems can generate symmetric coexisting attractors, which is called conditional symmetry, and chaotic flows with such conditional symmetry have been presented [Li et al., 2017]. He proposed a new chaotic flow with an infinite number of strange attractors [Li et al., 2017]. He described a periodically-forced system with an infinite number of coexisting limit cycles, attracting tori, and strange attractors [Sprott et al., 2017]. He proposed a chaotic flow, all of whose terms are quadratic except one which is linear and whose coefficient controls the amplitude and frequency of oscillations without inducing bifurcations [Li et al., 2017].
Sprott has examined simplifications of the Lorenz system and proposed a chaotic system with two fewer terms and a single parameter \cite{Sprott2009a}. Sun & Sprott \cite{Sun2006} and a fractional-order simplified Lorenz system \cite{Sun2010}. Hyperchaotic behavior of a new simplified Lorenz system with a sinusoidally-forced parameter and feedback control were investigated \cite{Sun2012}. He also investigated the fractal basin of the Lorenz model \cite{Djellit2012} and of a modified Lorenz system \cite{Djellit2012}. He examined the Lorenz system in a new region of parameter space \cite{Li2013} and found a previously unknown regime of multistability. He proposed a partial and complete piecewise-linear version of the Lorenz system which has an independent total amplitude control parameter \cite{Li2015}. The Lorenz system (with slightly changed parameters) exhibits tristability with two stable equilibria and one strange attractor, and its basins of attraction were examined using a 3D printer \cite{Kong2017}. Also, a modified Chua circuit with one, two, and three scrolls was introduced \cite{Elhadi2010}. He has shown that the Lorenz and Chen systems have the same attractor in $t \to \infty$ and $t \to -\infty$, respectively, and some of the behaviors of these two models which are equivalent have been investigated \cite{Sprott2015}. He studied the synchronization of two Rossler systems with a temporally-periodic coupling term \cite{Buscarino2017}.

2.1.3. Chaotic systems with complex variables

Complex-variable chaotic systems have been relatively less studied, but Sprott has identified the simplest nonautonomous \cite{Marshall2009} and autonomous \cite{Marshall2010} examples of such systems.

2.1.4. Nonautonomous chaotic systems

Externally driven systems generally have a more complex behavior and a higher-dimension attractor than an autonomous system with the same number of variables. Chaotic flows with a sinusoidal drive and a cubic nonlinearity \cite{Elshahib2010} and a signum nonlinearity \cite{Sun2014} have been examined. Also, the logistic map with a time-varying parameter has been discussed \cite{Mirus2008, Elhadi2008}.

2.1.5. Fractional-order chaotic systems

Fractional-order chaotic systems are of much current interest. Sprott investigated an electronic chaotic oscillator and a mechanical chaotic jerk model with fractional order $2 + \varepsilon$ where $0 < \varepsilon < 1$ \cite{Ahmad2003} as well as a fractional-order simplified Lorenz system \cite{Sun2010} and synchronized fractional-order systems \cite{Chen2012b, Chen2012a}.

2.1.6. Spatiotemporal chaos

Mathematical models in ecology often need to incorporate spatial dependence to accurately model real-world systems \cite{Wildenberg2006}. There are many studies of chaos in systems described by partial differential equations (PDEs). Chaotic behaviors of Lotka–Volterra models \cite{Sprott2005a, Vano2006} and their high-dimensional extensions \cite{Wildenberg2006a} have been studied. A spatiotemporal chaotic system with cyclic symmetry has been investigated \cite{Chlonverakis2009}. Sprott has discussed the Kuramoto–Sivashinsky equation as a simple chaotic partial differential equation and searched for even simpler examples \cite{Breunritt2004}.

2.1.7. Other chaotic systems

Delay in discrete and continuous systems can cause more complex behaviors \cite{Sprott2006a, Sprott2007}. In addition, Sprott studied the use of artificial neural networks for generating chaos \cite{Sprott1998}.

2.1.8. Bifurcations

Bifurcations are important in nonlinear dynamical systems, and Sprott has done much work in this area. He showed that the most probable route to chaos in high-dimensional artificial neural networks is the quasi-periodic route and that increasing the dimension of a system makes chaotic solutions more likely \cite{Albers1998}. He showed that the most probable route to chaos in high-order systems is through Neimark–Sacker bifurcations \cite{Albers2006}.

2.1.9. Basins of attraction

Complex dynamical systems can generate different attractors, each with a basin of attraction. Such systems are multistable, and they are important in...
the engineering applications since a small perturbation can cause a huge shift in the system’s state from one attractor to another. Sprott has investigated multistability in many systems such as the delayed Hénon map \cite{Sprott2006}. He has shown that symmetric chaotic flows such as the Rössler system have regimes of multistability not previously known \cite{Sprott2017}, and he has categorized basins of attractions into four classes depending on their size and extent \cite{Sprott2015}. Coexistence of some strange attractors and limit cycles in symmetric systems has been investigated \cite{Li2015a}.

2.1.10. Fractal art

A byproduct of his search for new chaotic maps and flows has been thousands of examples of fractal strange attractors and their basins of attraction, many of which have aesthetic appeal. Sprott has automated the search for visually interesting fractals and has published many of the most appealing ones along with the algorithms for their production \cite{Sprott1997, Sprott2003, Sprott2008, Sprott2004a, Taylor2004, Draves2004, Sprott1999, Sprott1999c, Chapman2003}. He was able to demonstrate a relation between the aesthetic appeal and their fractal dimension \cite{Sprott1993b, Aks2004, Sprott1996, Draves2008}. He had one of the first fractal galleries on the Web (http://sprott.physics.wisc.edu/fractals.htm), and for over 20 years a new “Fractal of the Day” has been regularly produced by his programs. He is currently writing a book, *Elegant Fractals: Automated Generation of Computer Art*, to be published in 2018.

2.1.11. Chaotic electrical circuits

Sprott has had a long interest in electronic circuits dating back to his teenage years as an amateur radio operator, and his first book was an electronics text \cite{Sprott1983}. Thus it was natural for him to propose many new examples of chaotic electrical circuits using only resistors, capacitors, diodes, and inverting operational amplifiers mostly based on the simple equations found in his search for new chaotic systems \cite{Sprott2000a, Sprott2000b, Kiers2004, Kiers2004b, Piper2006, Sprott2011d, Buscarino2010, Sprott2010}. He has been especially interested in finding the simplest circuits that oscillate chaotically.

2.1.12. Chaotic models


He also proposed a model of the world in which Newton’s first and second laws hold, but where the third law takes the form that the forces between any two objects are equal in magnitude and direction as might apply in biological systems such as a fox chasing a rabbit, and he found a rich variety of behaviors including chaos for the two-body problem in two spatial dimensions \cite{Sprott2006d}.

After modeling a complex system, estimating its parameters for different behaviors is desired. Sprott has proposed a new cost function to estimate parameters of chaotic systems with a statistical model of its phase space \cite{Shekotich2017}, and has provided a new viewpoint for the estimation of model parameters \cite{Jafari2014}.

2.2. Control and synchronization of chaotic systems

Chaos can be either a virtue or a vice, but in either case, it is useful to have some control over it, and many practical applications such as secure communications require the synchronization of two chaotic systems. Sprott’s interest in this problem arose from a desire to improve the confinement of plasmas by controlling their ubiquitous magnetic fluctuations. He proposed a control method using periodic parametric perturbations \cite{Mirus2004, Sprott1999c} and has also described a matrix controller.
for flows to allow a transition between asymptotically stable limit sets and hyperchaos [Elhadj & Sprott 2012]. Amplitude control of chaotic systems can be helpful in detecting multistability and hidden attractors [Li & Sprott 2014b, Li et al. 2016]. A method for amplitude control of chaotic systems via a control function has been proposed [Li & Sprott 2015]. He also presented a general method for amplitude control of chaotic systems by changing the degree of some terms in the governing equations [Li et al. 2015].

Chen posed a universal nonlinear control law for synchronization and anti-synchronization of chaotic systems [Maus & Sprott 2013]. He also discussed the synchronization of chaotic electronic circuits [Bai et al. 2002] and proposed a universal nonlinear control law for synchronization of 3D quadratic flows [Elhadj & Sprott 2012d]. A sliding mode control for synchronizing integer-order chaotic systems and a class of fractional-order chaotic system were also presented [Chen et al. 2012c]. Chen et al. [2012d] has proposed an adaptive modified hybrid function projective synchronization for chaotic and hyperchaotic systems with complex variables and parameters [Liu et al. 2014].

2.3. Time-series analysis

Like so many others, Sprott has been interested in finding evidence of chaos in real-world systems through the analysis of a time series of experimental data. He proposed a method based on singular-value decomposition to model a system using its time series [Rowlands & Sprott 1992]. He has improved the method for calculating the correlation dimension from measured data [Sprott & Rowlands 2001] and investigated the relation between the correlation dimension and the Kaplan–Yorke dimension [Chlouverakis & Sprott 2005]. He proposed a mechanism for detecting weak chaos in complex ecosystems [Sprott et al. 2005b]. He described a method to estimate the optimal time-delay embedding dimension [Maus & Sprott 2011] and Lyapunov exponent spectra [Maus & Sprott 2013] using an artificial neural network. Estimating Lyapunov exponents from a time series is an important problem since it can be used to predict tipping (bifurcation) points [Nazarimehr et al. 2017d]. He described a method for noise reduction in respiratory sounds [Abdelal et al. 2014] and proposed using the rate of divergence to differentiate four types of voice signals [Calawerts et al. 2015]. He also proposed a nonlinear model with exogenous inputs (NARX) based on a novel recurrent fuzzy function to predict a chaotic time series [Goudarzi et al. 2016].

3. Discussion

Only a small fraction of low-dimensional chaotic systems exhibit chaos. Professor Sprott has teased out many new examples of chaotic systems over the past three decades. He has published important books such as Chaos and Time-Series Analysis [Sprott 2003], and Elegant Chaos [Sprott 2003], and published many special chaotic systems such as those with hidden attractors and used a variety of tools to study their dynamics. He has modeled many real-world systems and explored methods for chaos control and synchronization. We thank Professor Sprott for his brilliant works over the years.

4. Congratulatory Messages to Professor Sprott

In the following, the authors provide some memories of their collaborations with Professor Sprott.

Fahimeh Nazarimehr: Chaos and nonlinear dynamics is a research field with many interesting problems. It is a great honor for me to learn from some of the best scientists in this area. I have collaborated with Professor Sprott and learned much from him. It is my pleasure to see him healthy and remain an enthusiastic pioneer in the field.

Sajad Jafari: During my PhD exam, I had no idea about working under the supervision of Professor Hashemi Golpayegani in the field of chaos and nonlinear dynamics. I was afraid of starting something new. However, it was a great luck of my life when he gave me some papers from Professor Sprott and told me to read these papers which are from a wise scientist. That was the beginning of my story with that kind Professor from whom I learned much and owe a lot. I wish him health and happiness forever.

Guanrong Chen: Before I met Clint in Hong Kong, I knew he was excellent in chaotic time-series analysis from his popular book Chaos and Time-Series Analysis. Through the years after meeting him, I gradually and surprisingly found that he


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is superb in constructing and searching for new kinds of chaotic attractors from lower-dimensional, simply-structured autonomous systems, with some unusual and amazing properties that are unexplained by modern mathematical chaos theory. I consider him an asset to our chaos community.

Tomass Kapitaniak: Clint proposed a number of simple looking, low-dimensional dynamical systems. With these examples, dynamics look simple, strange attractors are not too strange, and hidden attractors are not too deeply hidden, i.e. easy to uncover.

Gennady A. Leonov and Nikolay V. Kuznetsov: The elegant and meaningful examples of dynamic systems proposed by Professor Sprott are opening the veil of the mystery of chaos and motivate us to develop new analytical methods for the investigation of chaotic dynamics.

Chunbiao Li: I feel that chaos and multistability is MAGIC when I am working with Professor Sprott. The year of 2012 was a great period in my life since it brought me to work with him. During my time in Madison, we discussed chaos almost every day, and sometimes we played tennis. I wrote a poem to describe that wonderful time. “We played tennis under the cloud; we discussed academic questions under the rain; we said goodbye day after day, but your figure is in my memory every day. I am a visiting scholar from abroad. While I am in your sun, all in all, I hope the days continue and forever be there; I hope the sky and cloud bring my best wishes to you everywhere.”

Zhouchao Wei: First of all, I, together with my friends, wish to send our best wishes to Professor Sprott on the occasion of his 75th birthday. I have known Professor Sprott since my first year as a graduate student at Guangzhou in the fall of 2007. Professor Sprott has made great contributions to chaos research. He was always encouraging, even when we did not agree. (He was much more rigorous than me.) He was a true mentor, and I am grateful for the experience and all the ideas he provided me.

References


Elhadj, Z. & Sprott, J. C. [2013e] “Transformation of
Elhadj, Z. & Sprott, J. C. [2013a] “A rigorous deter-
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Ghomashchi, H., Esteki, A., Sprott, J. C. & Nasrabadi,
Goudarzi, S., Jafari, S., Moradi, M. H. & Sprott, J. C.
Hens, C. R., Banerjee, R., Feudel, U. & Dana, S. K.
time-reversible chaos for Gibbs’ canonical oscillator,”
oscillators near and far from equilibrium,” Mol. Simul. 42, 1300–1316.
and the limits of predictability for the solar-wind-driven
with a line equilibrium.” Chaos Solit. Fract. 57, 79–84.
chaotic flows with a line equilibrium" [chaos, solitons
Jafari, S., Nazarimehr, F., Sprott, J. C. & Golpayegani,
chaotic flow with a plane of equilibria,” Int. J. Bifurcation and Chaos 26, 1650098-1–6.
Kiers, K., Schmidt, D. & Sprott, J. C. [2004b] “Preci-


