The RFP: Plasma Confinement with a Reversed Twist

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A tutorial on the Reversed Field Pinch (RFP).

- Worldwide research on the RFP has been ongoing since 1960’s when a “quiet period” observed in ZETA device plasmas was correlated with reversed toroidal field at the edge of the plasma.

- Although the integrated effort in RFP research is much smaller than for the tokamak or stellarator, the RFP program continues to make progress in key areas important to establishing the viability of the concept for fusion energy.

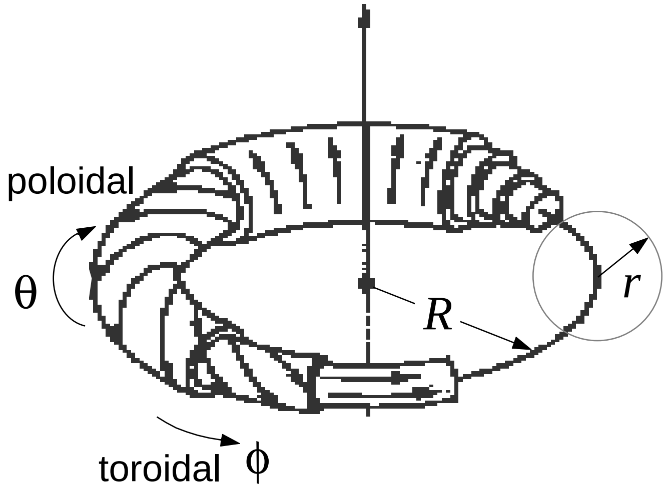
- This tutorial will focus on answering “What’s an RFP?”
  – advantages as a fusion concept
  – important physics results

Key theme: Role of magnetic turbulence in plasmas
  – a laboratory in nonlinear MHD
  – magnetic dynamo
  – transport from magnetic fluctuations
  – reducing transport using physics understanding

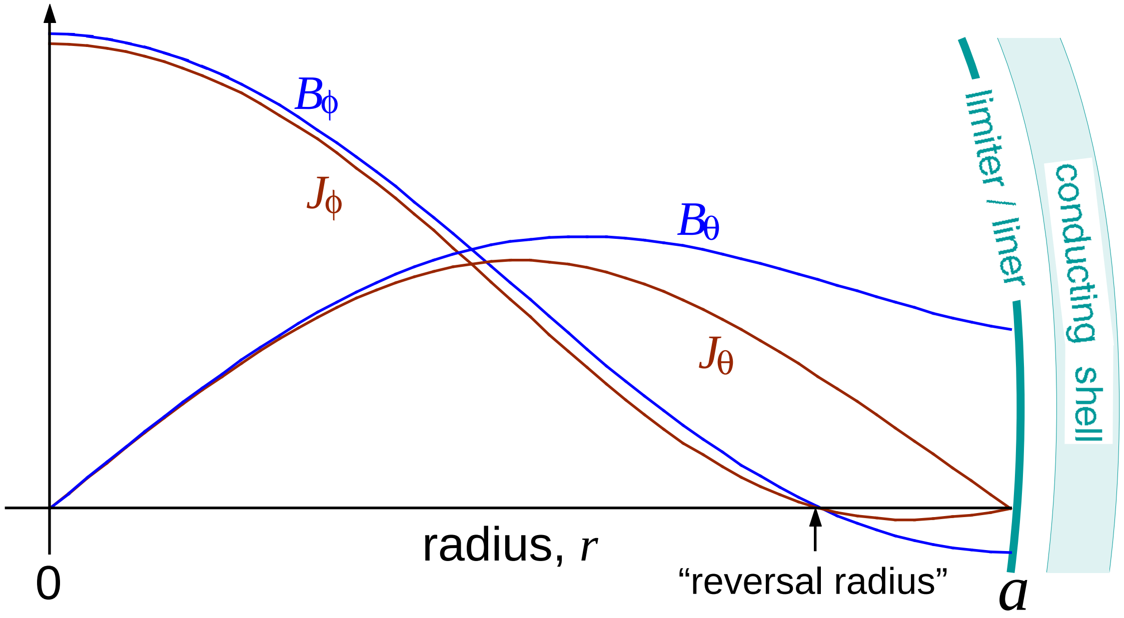
RFP magnetic equilibrium has large shear with toroidal field, B_ϕ ≈ poloidal field, B_θ .

- Toroidally axisymmetric, current-carrying plasma:

  (r, θ, ϕ) coordinates
  R = major radial

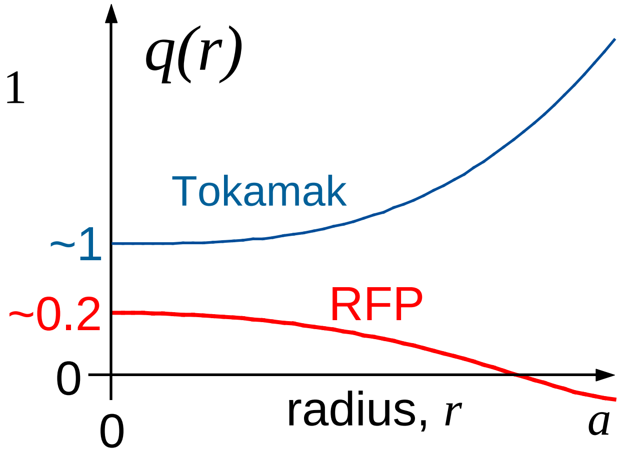


- Diffuse field and current:



- Safety factor, $q(r) = \frac{rB_\phi}{RB_\theta} = \frac{d\phi}{d\theta} << 1$

- Large magnetic shear, $\frac{1}{q}\frac{dq}{dr}$



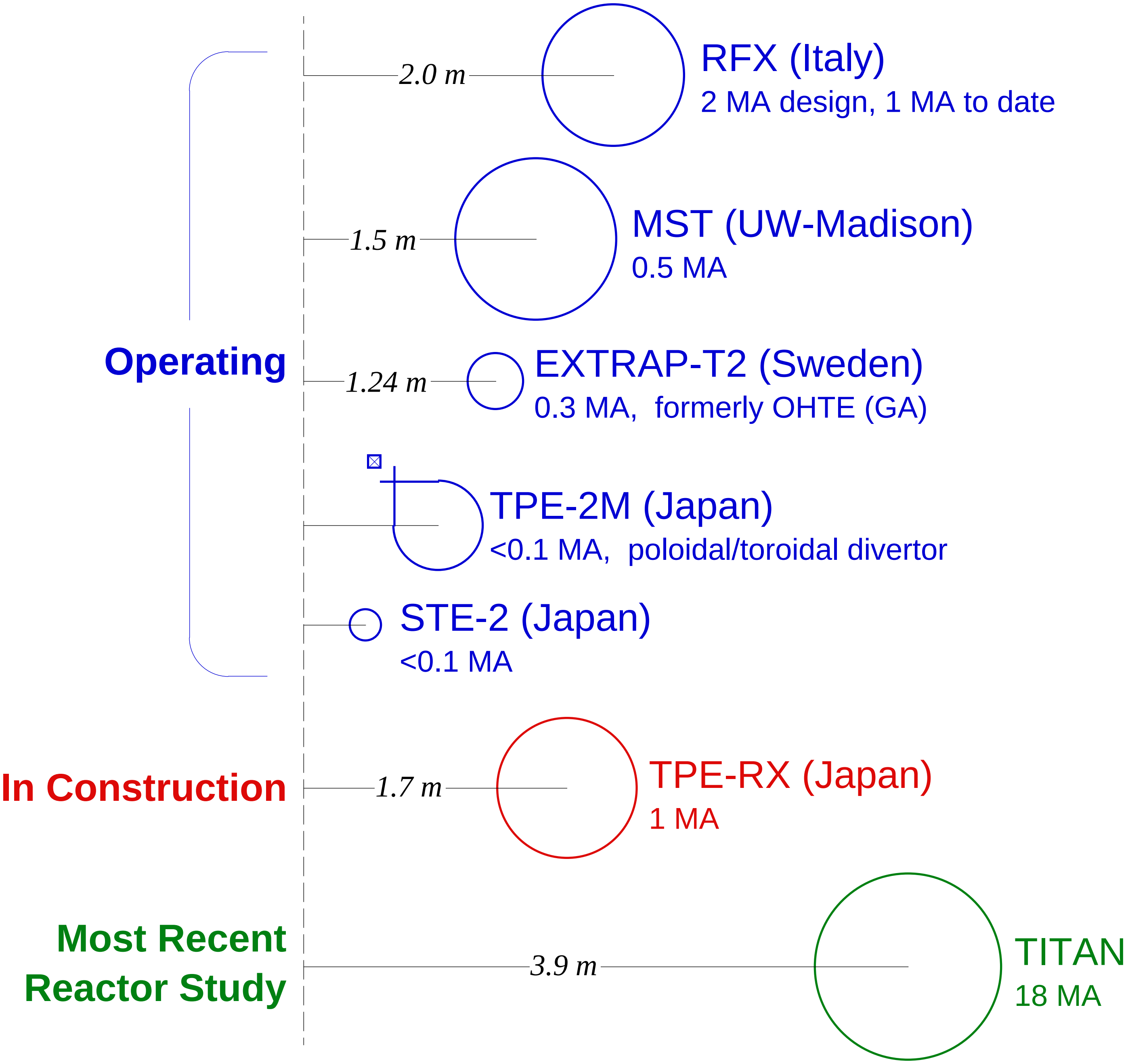
Smaller magnetic field makes the RFP conceptually compact, high power density fusion reactor.

Key RFP reactor attributes:

- compact
- high beta β~10%
- high “engineering” beta (low field at plasma surface)
- low magnet forces, non-superconducting construction
- current disruption-free operation; “soft” density limit
- free choice of aspect ratio R/a

Major Physics & Engineering Challenges:	*Relative* *Emphasis*
• improving confinement & beta	*1*
• conducting shell stabilization	*0.1*
• large current drive requirement	*0.05*
• large wall loading (high power density)	*0.01*

Modest international RFP program



Operating

RFX (Italy)
2 MA design, 1 MA to date

MST (UW-Madison)
0.5 MA

EXTRAP-T2 (Sweden)
0.3 MA, formerly OHTE (GA)

TPE-2M (Japan)
<0.1 MA, poloidal/toroidal divertor

STE-2 (Japan)
<0.1 MA

In Construction

TPE-RX (Japan)
1 MA

Most Recent Reactor Study

TITAN
18 MA

- **Past medium sized experiments:**
  HBTX Series (Culham), ZT-40M (LANL), TPE Series (Japan),
  OHTE (GA), ETA-BETA-II (Italy), REPUTE (Japan)

Toroidal field reversal guarantees large shear necessary for ideal MHD stability.

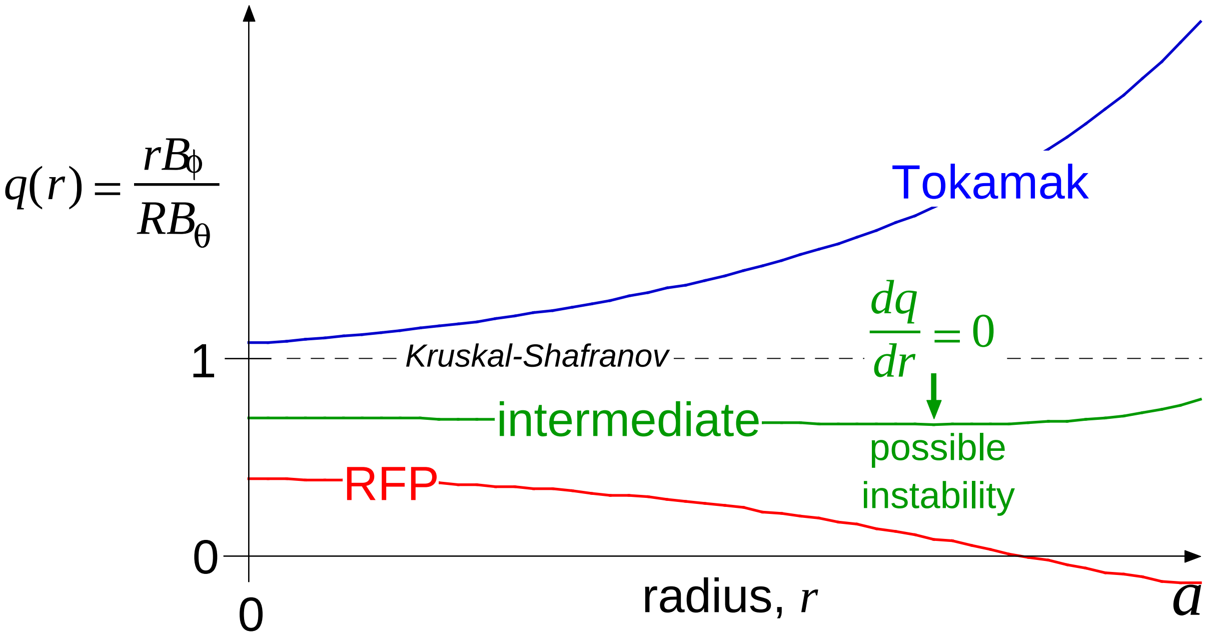
- Ideal kink stability generally requires either no shear minima or $q > 1$ (Kruskal-Shafranov).
- Ideal interchange stability requires finite shear for $q < 1$:

$$\frac{dp}{dr}(1-q^2)+\frac{rB_\phi^2}{32\pi}\left(\frac{1}{q}\frac{dq}{dr}\right)^2 > 0 \qquad \text{(Mercier criterion)}$$

destabilizing peaked pressure

weak toroidicity in RFP (bad curvature)

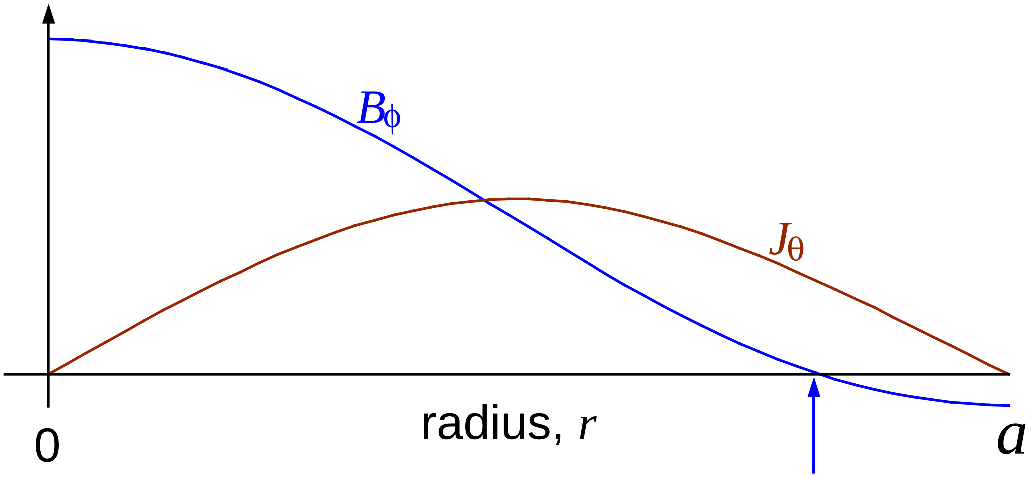
stabilizing magnetic shear



- Ideal MHD beta limit ~40% < observed beta 10-20%

An apparent mystery: how is an RFP sustained?

- A resistive, steady-state, axisymmetric reversed toroidal field equilibrium is not possible :



reversal implies: $$(\nabla \times B)_\theta = \cancel{\frac{1}{R}\frac{\partial B_r}{\partial \phi}} - \frac{\partial B_\phi}{\partial r} \neq 0$$

$\Rightarrow J_\theta \neq 0$, which for finite η

$$\Rightarrow E_\theta = -\frac{\partial}{\partial t}\int B_\phi \, dS \neq 0$$

(non steady-state)

- But in an RFP, field reversal is maintained as long as toroidal current is sustained ($\langle E_\theta \rangle = 0$) :

$\Rightarrow$ non-inductive poloidal current

$\Rightarrow$ symmetry breaking turbulence

} The RFP dynamo

RFP seeks “relaxed” minimum energy state.

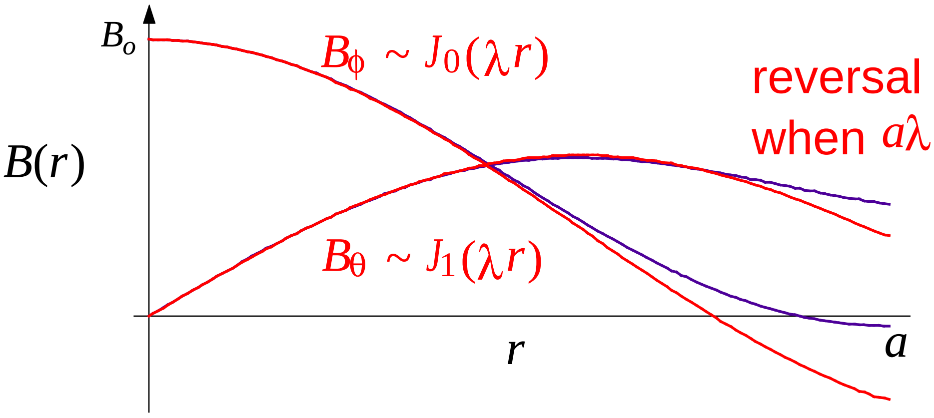
- A *relaxed state* is the minimum energy configuration of a finite resistance plasma subject to the *global* constraint of constant magnetic helcity [J.B. Taylor, PRL 33, 1139 (1974)].

magnetic helicity: $K = \int_V \mathbf{A} \cdot \mathbf{B}\, dV$ = constant of motion

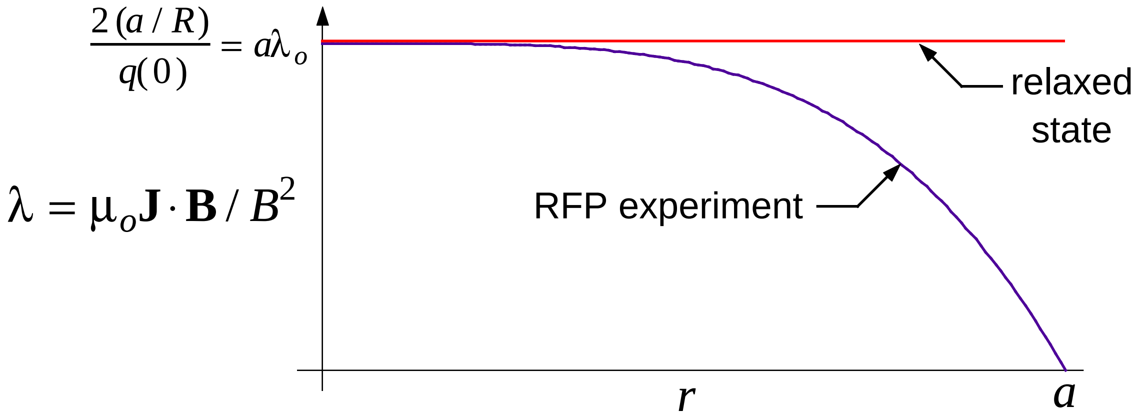
- For a pressureless (force-free) plasma within a closed, perfectly conducting shell, relaxed states are :

$$\nabla \times \mathbf{B} = \lambda \mathbf{B} \quad \text{where} \quad \lambda = \mu_o \mathbf{J} \cdot \mathbf{B} / B^2 = \text{constant}$$

in a cylinder: $B_\phi = B_o J_o(\lambda r) \quad B_\theta = B_o J_1(\lambda r)$



reversal occurs when $a\lambda > 2.404$

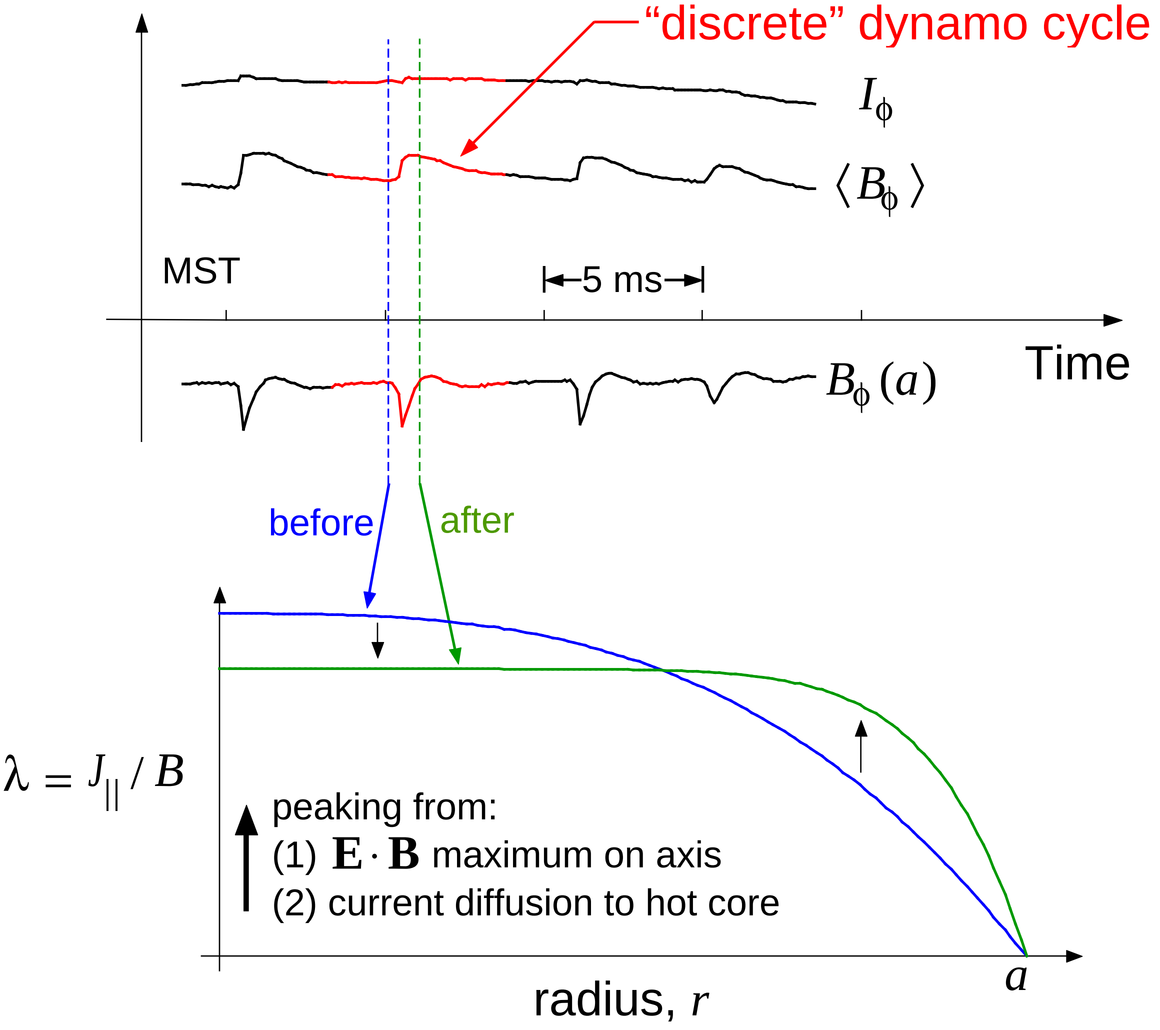


relaxed state

RFP experiment

“Discrete event” dynamo illustrates tendency toward relaxed state.

- Sudden relaxation events result in toroidal flux generation



peaking from:
(1) $\mathbf{E} \cdot \mathbf{B}$ maximum on axis
(2) current diffusion to hot core

Resistive MHD provides a detailed theory for the RFP dynamo mechanism.

- Essential behavior captured in pressureless limit

$$\frac{\partial \mathbf{A}}{\partial t} = S\mathbf{V}\times\mathbf{B} - \eta\mathbf{J} \quad \text{(Ohm's law)}$$

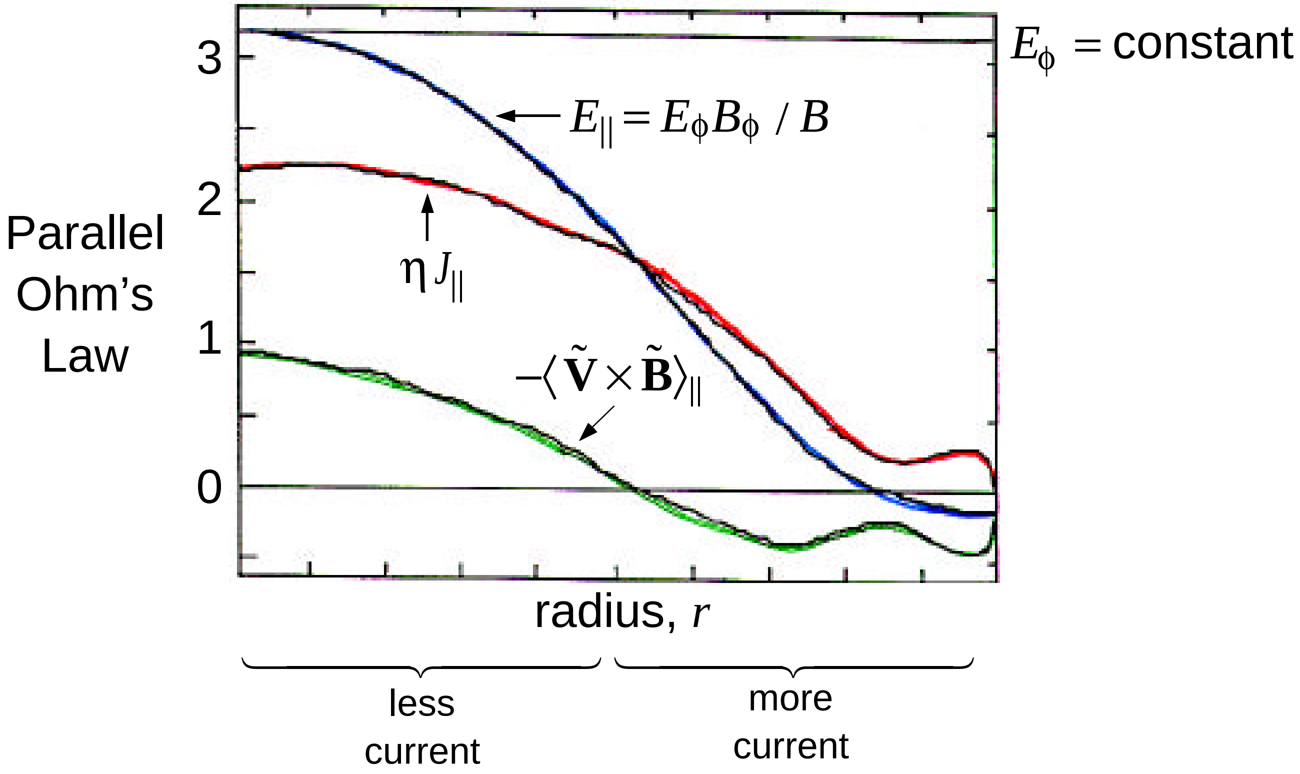
$$\frac{\partial \mathbf{V}}{\partial t} = -S\mathbf{V}\cdot\nabla\mathbf{V} + S\mathbf{J}\times\mathbf{B} + \nu\nabla^2\mathbf{V}$$

Key MHD parameter
Lundquist number:

$$S = \tau_{res} / \tau_{Alfven}$$

- Turbulent $\langle \tilde{\mathbf{V}}\times\tilde{\mathbf{B}}\rangle_{\|} = \eta J_{\|} - E_{|}$ sustains J_θ required for $\langle B_\phi \rangle > 0$:
  - essential for conventional, Ohmically driven RFP.
  - converts poloidal flux into toroidal flux.

- Nonlinear, resistive MHD computational demonstration of RFP sustainment :



Resistive MHD tearing modes are the dominant turbulence in the RFP.

- Tearing modes $\mathbf{k} = \frac{m}{r}\hat{\theta} + \frac{n}{R}\hat{\phi}$ driven by $\frac{d\lambda}{dr}$ at

  $q=m/n$ resonant surfaces ($\mathbf{k}\cdot\mathbf{B}=0$).

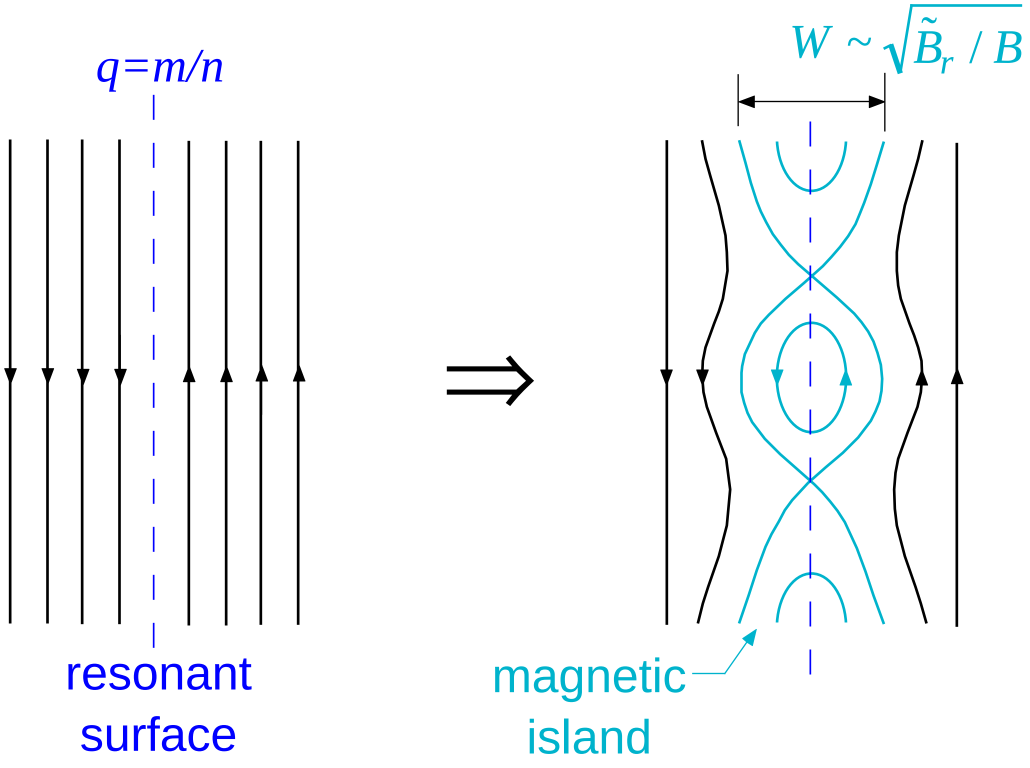
  growth rate: $\gamma \sim S^{\alpha}\,[\tilde{B}_r' / B_r]^{\beta}_{jump}$

  “jump” in the 1st derivative across resonant surface

  $$\tilde{B}_r'' + \left(\frac{F''}{F} + k^2\right)\tilde{B}_r \approx 0$$

  $$F \equiv \mathbf{k}\cdot\mathbf{B} \qquad \frac{F''}{F} \sim \frac{B''}{B} \sim \frac{d}{dr}\left(\frac{J_{\parallel}}{B}\right) = \frac{d\lambda}{dr}$$

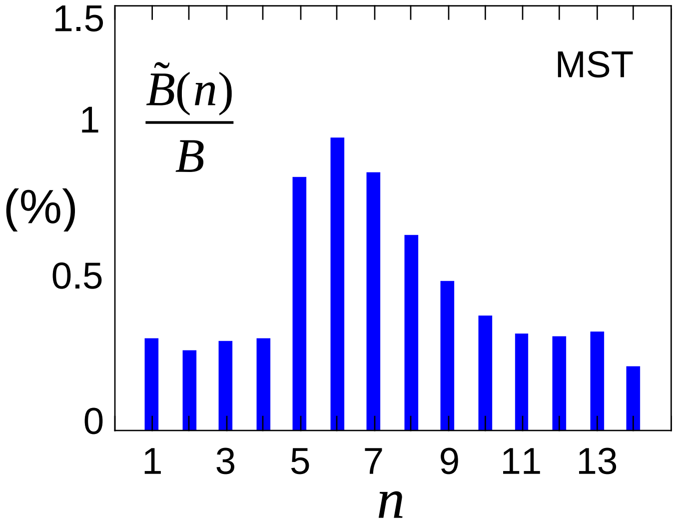
- “Tearing” refers to the reconnection of magnetic field lines allowed by finite resistivity.
  - changes magnetic field topology, exactly what is required for the RFP dynamo



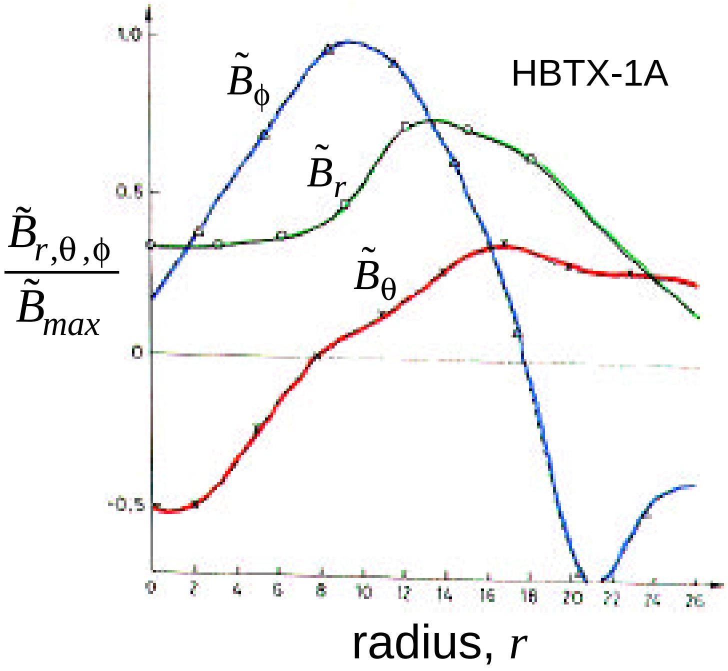
Although mode amplitudes only ~1%, island overlap produces global stochasticity.

- Mode amplitudes, spectrum, eigen-structure, etc. agree well with theory.

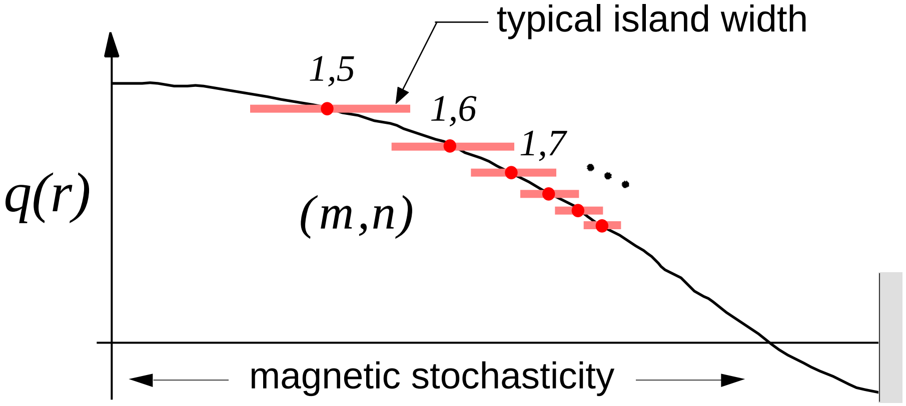
n-spectrum



Radial mode structure



- Close spacing of resonant surfaces encourages island overlap
  ⇒ global stochasticity.



Energy transport in the RFP results from magnetic fluctuations.

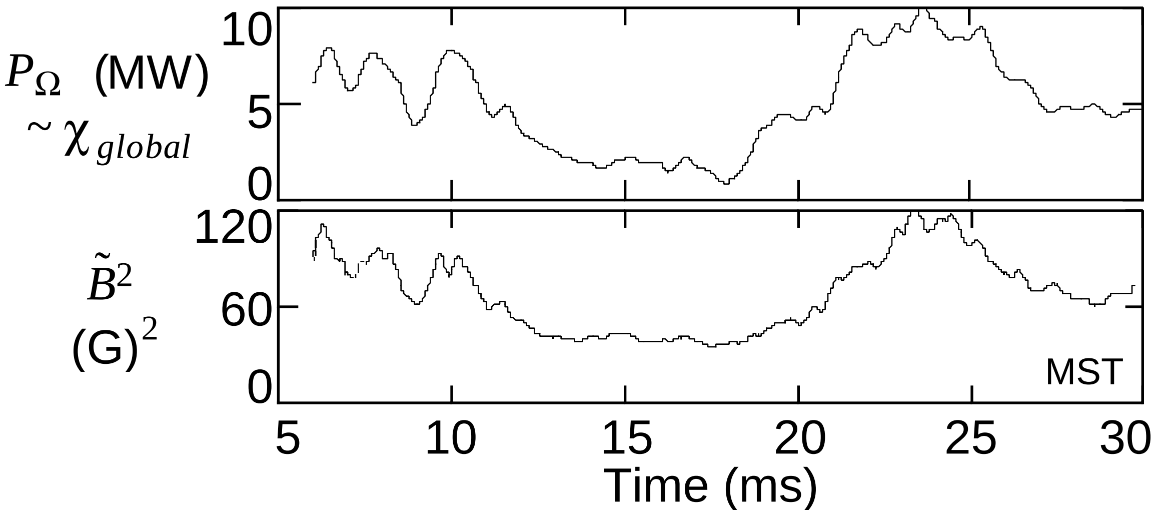
- Well known expectation for transport in a stationary, stochastic magnetic field (Rechester & Rosenbluth,1978)

$$\chi \sim v_T L_c (\tilde{B}_r / B)^2$$

$v_T =$ thermal velocity

$L_c =$ parallel $\tilde{B}$ correlation length

- Global power loss correlates strongly with fluctuation amplitude



Closer examination finds surprises in nature of transport.

- Generally, magnetic-fluctuation-induced radial fluxes from parallel motion are :

particles: $\Gamma_r = \langle n\mathbf{V}_{\|} \cdot \hat{\mathbf{r}} \rangle = \dfrac{\langle J_{\|}\mathbf{B} \cdot \hat{\mathbf{r}} \rangle}{qB} = \dfrac{\langle \tilde{J}_{\|}\tilde{B}_r \rangle}{qB}$

heat: $Q_r = \langle \mathbf{Q}_{\|} \cdot \hat{\mathbf{r}} \rangle = \dfrac{\langle Q_{\|}\mathbf{B} \cdot \hat{\mathbf{r}} \rangle}{B} = \dfrac{\langle \tilde{Q}_{\|}\tilde{B}_r \rangle}{B}$ $\left(\tilde{Q}_{\|} = \int \tfrac{1}{2} m v^2 v_{\|} \tilde{f}(v) dv \right)$

- Probe measurements of $\langle \tilde{J}_{|}\tilde{B}_r \rangle$ and $\langle \tilde{Q}_{|}\tilde{B}_r \rangle$ identify expected magnetic-fluctuation-induced transport

…but with surprises: $Q_e \approx \tfrac{3}{2} T_e \Gamma_e$ (convective transport)

$\Gamma_e, Q_e \sim v_{Ti}(\tilde{B}_r / B)^2$ (ambipolar constrained)

convective, ambipolar magnetic transport can occur when local magnetic fluctuation is dominantly generated by modes resonant at distant radii (P.W. Terry et al.)

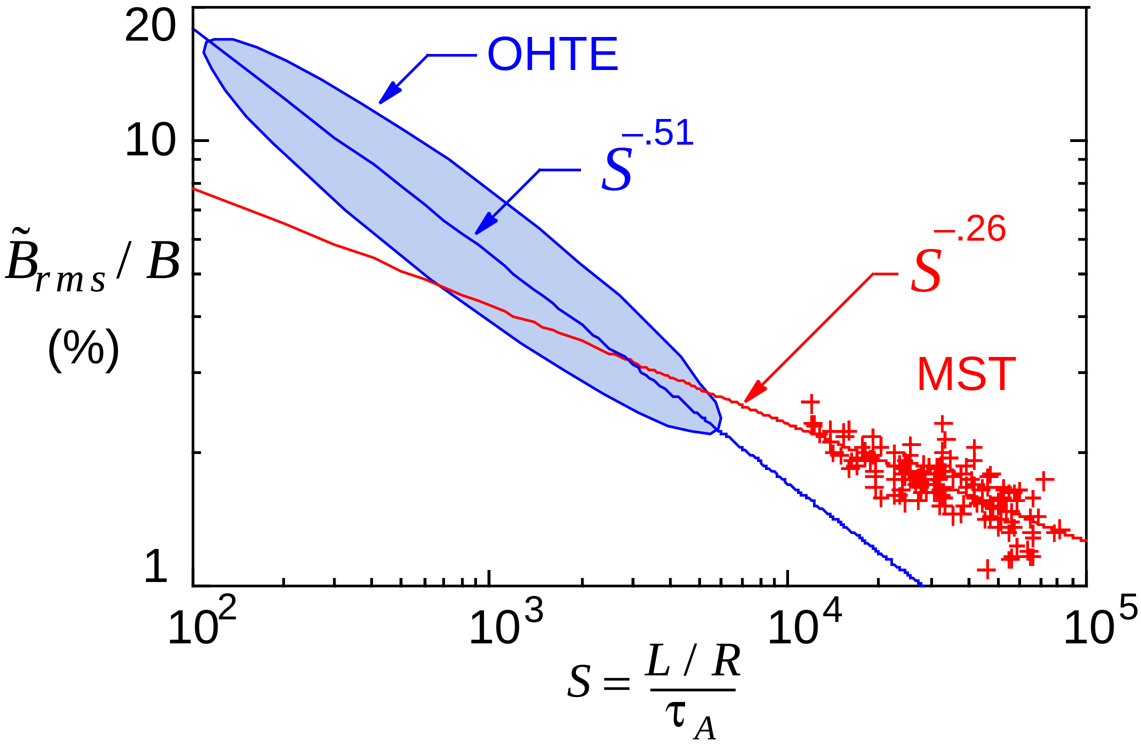
Controlled reduction of magnetic fluctuations best path to improved RFP confinement.

- Possible ways to reduce tearing fluctuations:
  - – natural S-scaling (larger, hotter plasmas have smaller $\tilde{B}$)
  - – active current profile $J(r)$-control (attack free energy)
  - – active feedback stabilization
  - – sheared flow (some experimental evidence)

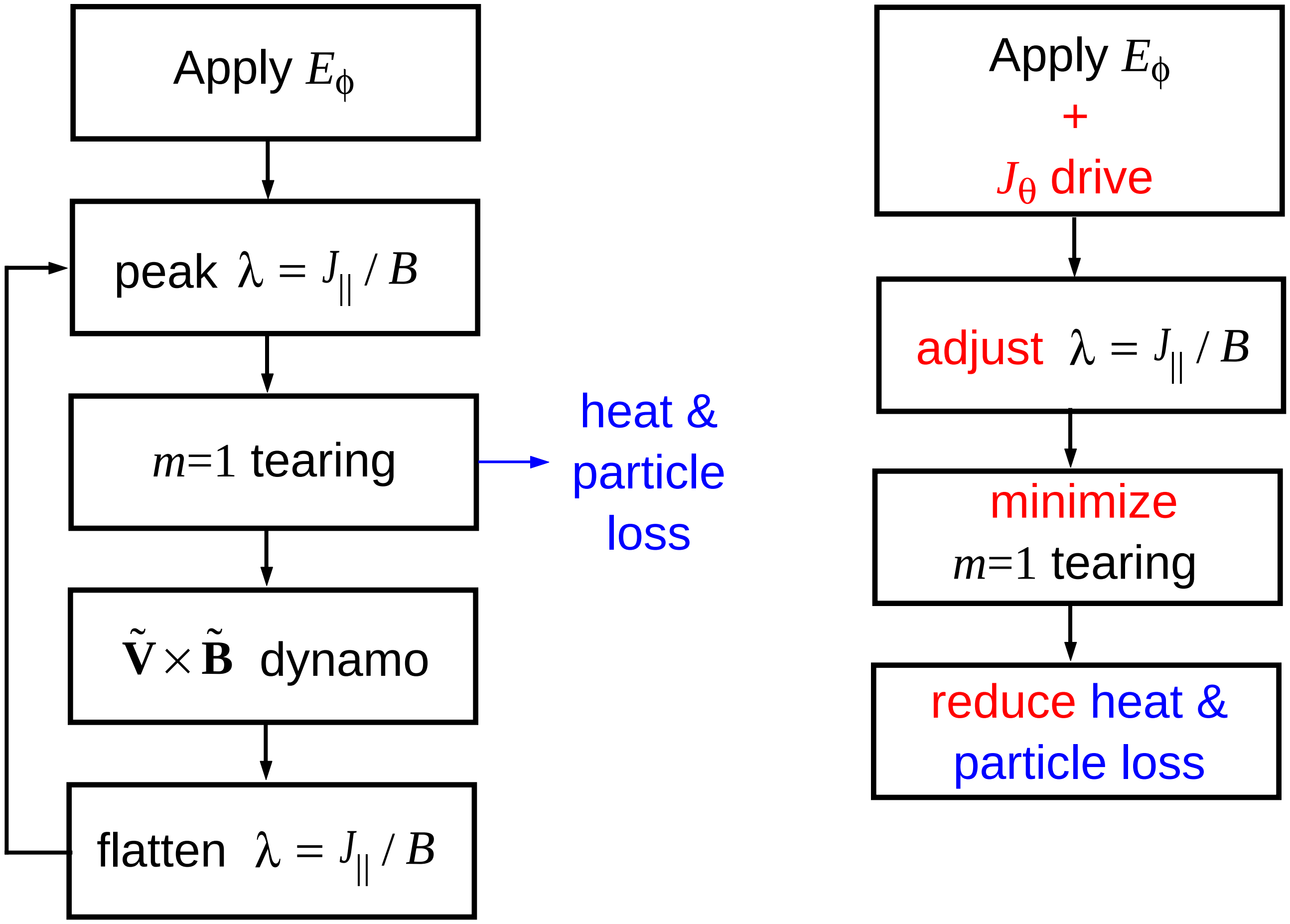
- Natural S-scaling appears weaker at larger $S = \tau_{res} / \tau_{Alfve}$:
  - – best possible scaling if $\tilde{B} \sim \tilde{V} \sim S^{-1/2}$ & Phase$(\tilde{V}, \tilde{B}) \sim S^0$

$$E_{||} + \langle \tilde{\mathbf{V}} \times \tilde{\mathbf{B}} \rangle_{||} = S^{-1} J_{||} \quad \text{(dimensionless Ohm's law)}$$

  - – weak scaling implies active fluctuation control



Current profile control to reduce dynamo.



Conventional RFP

(MHD dynamo)

Apply E_ϕ

peak $\lambda = J_{||} / B$

m=1 tearing

heat & particle loss

$\tilde{\mathbf{V}} \times \tilde{\mathbf{B}}$ dynamo

flatten $\lambda = J_{||} / B$

J(r)-controlled RFP

(reduce dynamo)

Apply E_ϕ + J_θ drive

adjust $\lambda = J_{||} / B$

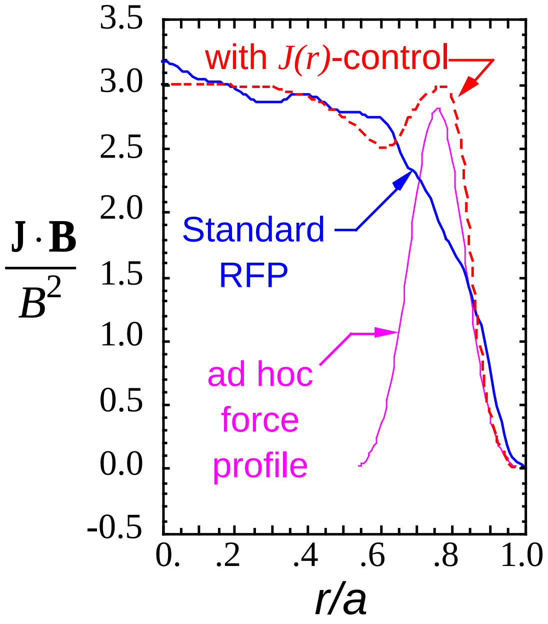
minimize m=1 tearing

reduce heat & particle loss

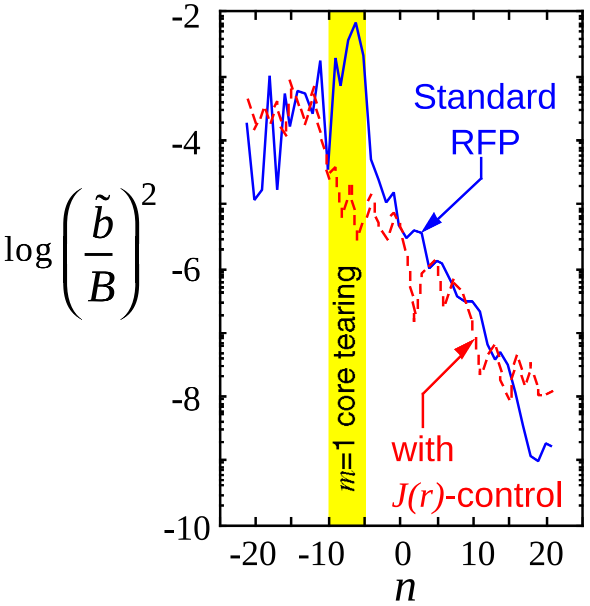
MHD Simulated Current Profile Control Demonstrates $\tilde{B}$ Suppression

- Ad hoc parallel force added to Ohm's law to simulate generic poloidal current drive :

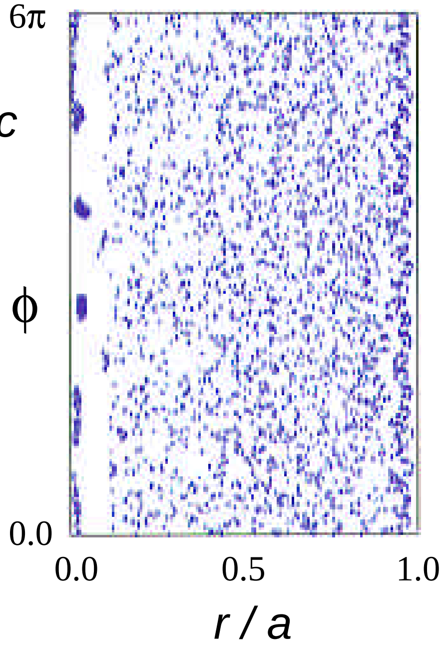
$$\frac{\partial \mathbf{A}}{\partial t} + \frac{F_a \hat{\mathbf{b}}}{ne} = \mathbf{V} \times \mathbf{B} - \eta \mathbf{J}$$



Nonlinear, 3D resistive MHD computation

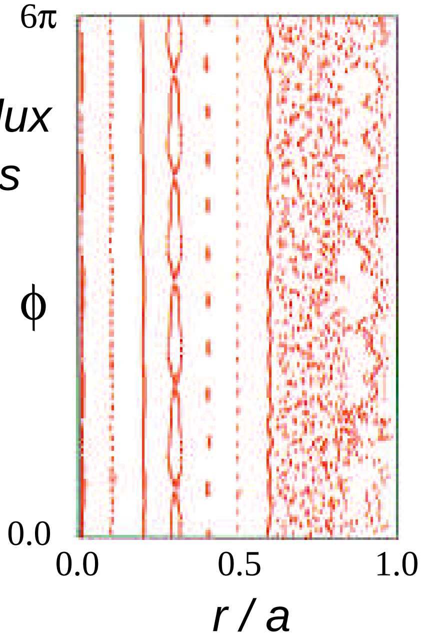


Standard RFP



Stochastic Field

RFP with J(r)-Control

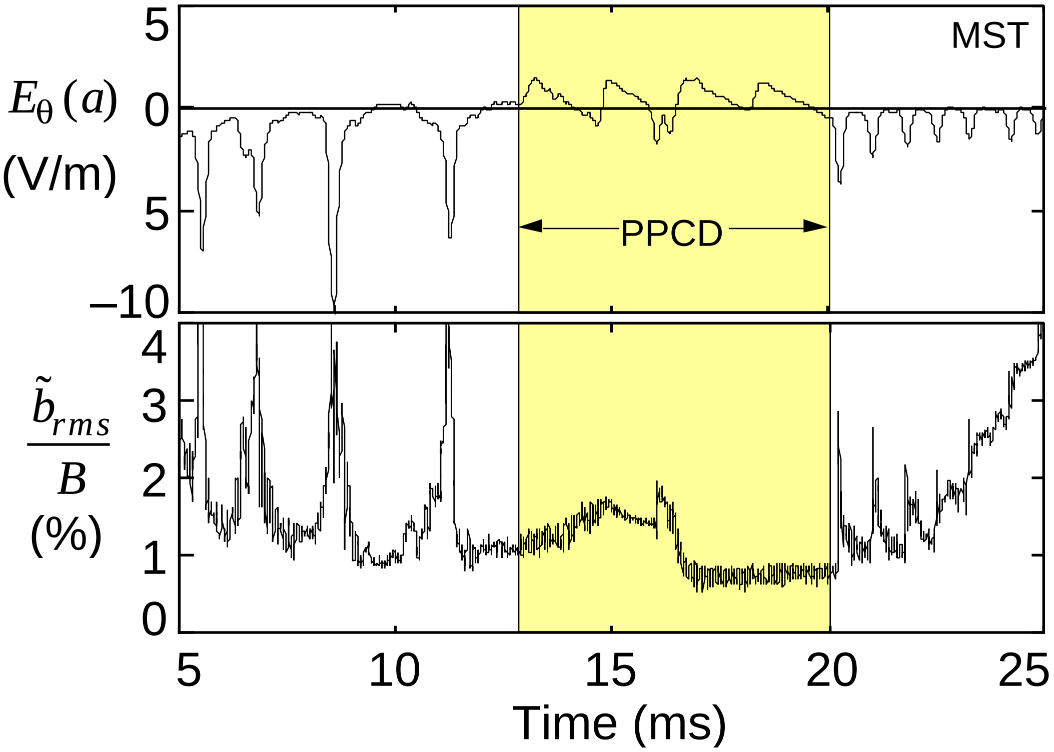


Closed Flux Surfaces

Inductive $J(r)$-control improves energy confinement five-fold in MST.

- Methods for $J(r)$-control:
  - – inductive pulsed poloidal current drive (PPCD)
  - – electrostatic current injection (in progress)
  - – RF current drive (future)

		Conventional RFP		PPCD
Fluctuation,	$\frac{\tilde{b}_{rms}}{B}$	1.5%	$\xrightarrow{\times\ 0.5}$	0.8%
Energy Confinement,	τ_E	1 ms	$\xrightarrow{\times\ 5}$	5 ms
Poloidal Beta,	β_θ	6%	$\xrightarrow{\times\ 1.5}$	9%



Apply poloidal E_θ to drive edge current.

Reduces Magnetic Fluctuation

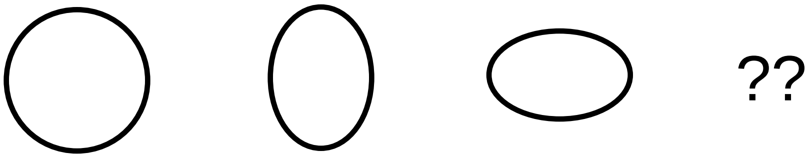
What’s the MHD optimized RFP?

- Current profile:
  - steady-state via electrostatic and RF current drive are or will be tested.

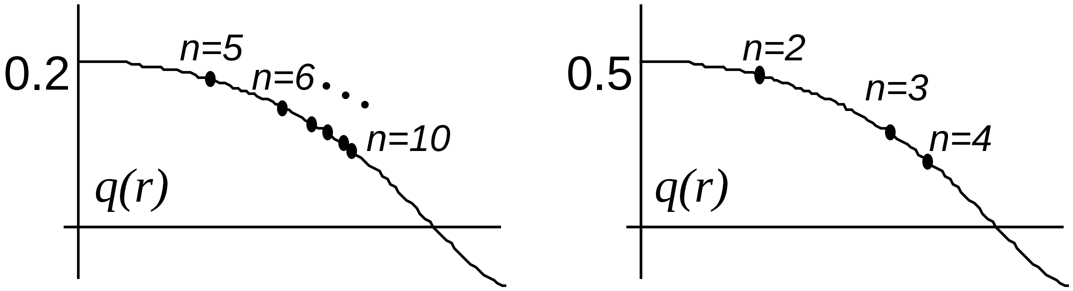
- Pressure profile:
  - need auxiliary heating to test beta limit (separate from transport)
  - with improved energy confinement from current profile control, pressure is increasing ⇒ pressure profile control

- Shape and aspect ratio:
  - all but a couple of RFPs have been circular toroids
  - all RFPs have $R/a \geq 3$; at small aspect ratio, there are fewer resonant modes ⇒ possibly better confinement or easier active control.

shape:



aspect ratio:



Summary

- RFP contributions to plasma science wide ranging:
  - practical fusion power
  - terrestrial laboratory for magnetic plasma dynamo
  - behavior of plasmas with strong magnetic turbulence

- Practical fusion potential stems from low field, high beta:
  - compact
  - low magnet forces, non-superconducting
  - disruption-free operation, & free choice of aspect ratio

- Conventional RFP requires a magnetic dynamo:
  - nonlinear, resistive MHD success story
  - consequent strong magnetic turbulence

- Understanding of fundamental processes reverses stigma "relaxation = poor confinement"
  - current profile control to circumvent dynamo
  - preserves and enhances attractive reactor features

Online RFP bibliography, including RFP reviews:

http://sprott.physics.wisc.edu/rfp/bib.htm

or

search: "Madison Symmetric Torus"