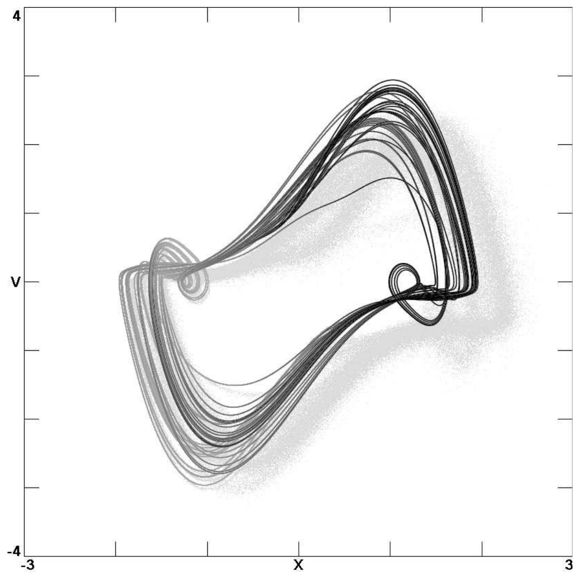

Fig. 6.5 Attractor for two identical coupled van der Pol oscillators in Eq. (6.9) with $b = 2.2, k = 0.7$, and initial conditions $(x_0, v_0, y_0, u_0) = (0, 1, 0, 1.7)$, $\lambda = 0.0829$.

6.3.3 *Coupled complex oscillators*

Any of the nonlinear complex oscillators in Table 2.5 can be coupled in a similar fashion to produce chaos. For example, the dissipative system

$$\begin{aligned} \dot{z}_1 - z_1^2 - \overline{z_1} + 1 &= -kz_2 \\ \dot{z}_2 - z_2^2 - \overline{z_2} + 1 &= -kz_1 \end{aligned} \tag{6.10}$$

is chaotic for $k = 2.5$ with an attractor as shown in Fig. 6.6.

6.3.4 *Other coupled nonlinear oscillators*

Any of the other acceleration-forced systems in Chapter 2 can be mutually coupled in a manner similar to the coupled pendulums and the coupled complex oscillators. Table 6.3 gives a few such examples, all of which are conservative with their corresponding state space plots shown in Fig. 6.7.

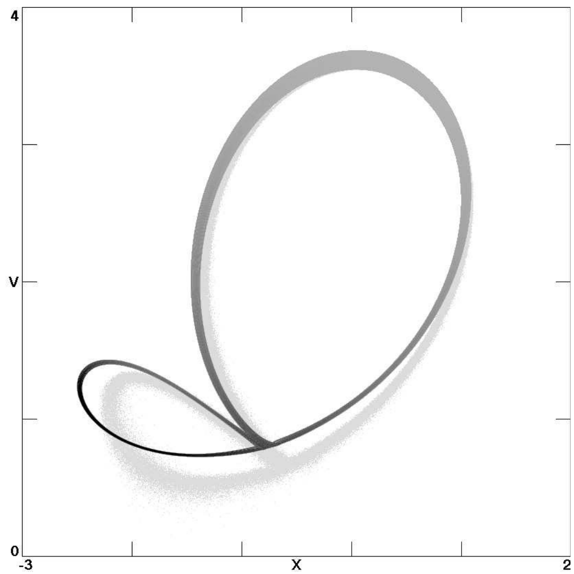

Fig. 6.6 Attractor for the coupled complex oscillators in Eq. (6.10) with $k = 2.5$ and initial conditions $(z_{10}, z_{20}) = (-2.4 + i, 1 - 3i)$, $\lambda = 0.0801$.

Table 6.3 Some chaotic coupled nonlinear oscillators.

Model	Equations	x_0, v_0, y_0, u_0	Lyapunov Exponents
CO_0	$\ddot{x} + \sin x = y - x, \ddot{y} + \sin y = x - y$	$3.9, 0.3, 0.5, 0.3$	$0.0057, 0, 0, -0.0057$
CO_1	$\ddot{x} + \sin x = y - x, \ddot{y} + \text{ sgn } y = x - y$	$-0.2, 0.4, -2, -0.3$	$0.0216, 0, 0, -0.0216$
CO_2	$\ddot{x} + \sin x = y - x, \ddot{y} + y^3 = x - y$	$1.2, 1.9, 0.4, -1.2$	$0.0645, 0, 0, -0.0645$
CO_3	$\ddot{x} + \text{ sgn } x = y - x, \ddot{y} + \text{ sgn } y = x - y$	$0.8, 0.8, -2.1, 1.6$	$0.0557, 0, 0, -0.0557$
CO_4	$\ddot{x} + \text{ sgn } x = y - x, \ddot{y} + y^3 = x - y$	$-0.4, 0.5, -0.7, 1.3$	$0.0796, 0, 0, -0.0796$
CO_5	$\ddot{x} + x^3 = y - x, \ddot{y} + y^3 = x - y$	$1.7, 1.1, -3.2, 1.1$	$0.1447, 0, 0, -0.1447$

6.4 Hamiltonian Systems

Many conservative systems including those above can be derived from Hamilton's equations

$$\begin{aligned} \dot{x} &= \tfrac{\partial H}{\partial v} \\ \dot{v} &= -\tfrac{\partial H}{\partial x} \end{aligned} \tag{6.11}$$

(and similar equations for $(\dot{y}, \dot{u})$, etc.), where the *Hamiltonian function* $H = H(x, v, y, u, \ldots)$ does not depend explicitly on time and is thus a